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Topological states in a microscopic model of interacting fermions

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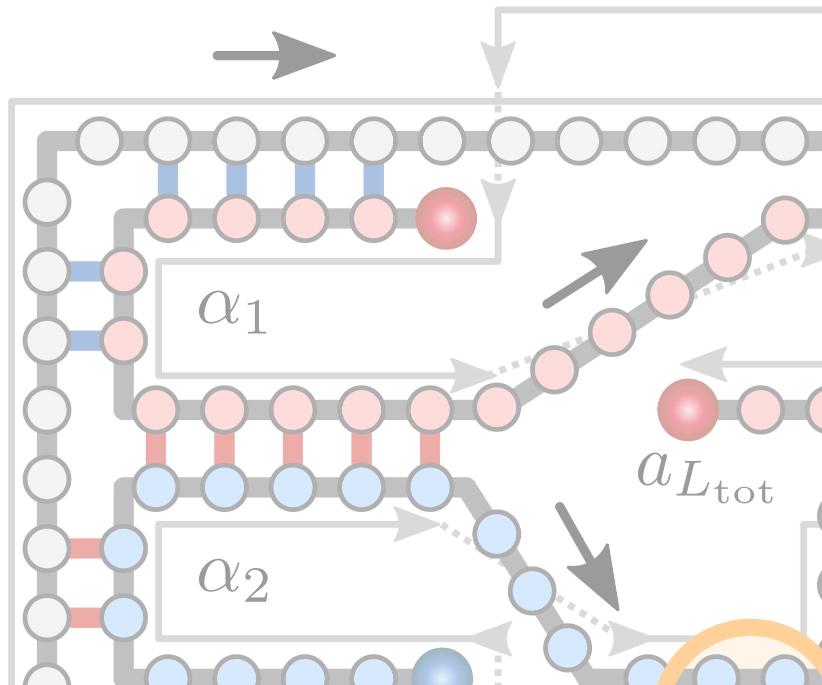
B8



CO.CO.MAT
Control of Quantum Correlations in Tailored Matter
SFB/TRR 21 - Stuttgart, Ulm, Tübingen

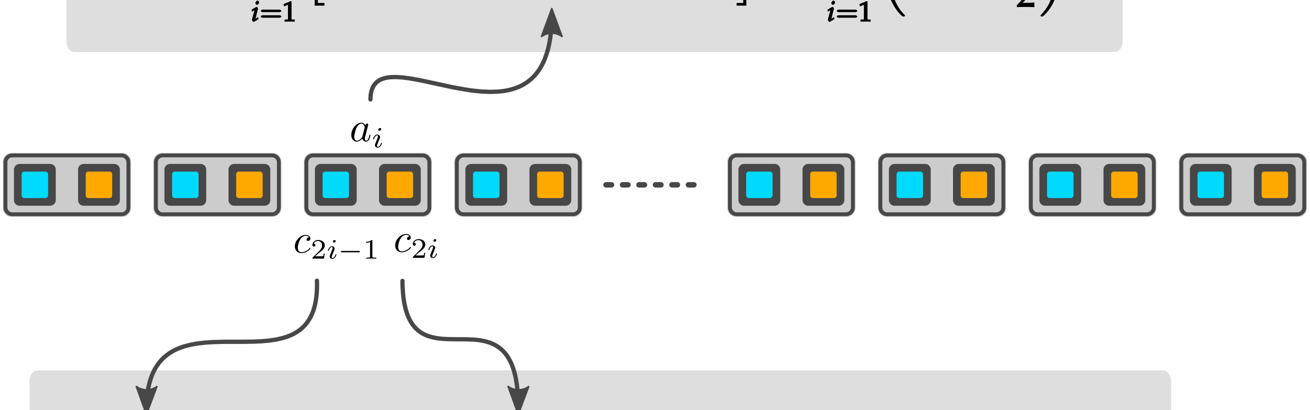
Motivation

The Majorana Chain

The Majorana Chain # Setting

Open chain of spinless fermions:
Nearest-neighbour hopping + Pairing + Chemical potential

$$H = -\sum_{i=1}^{L-1}\left[w a_i^\dagger a_{i+1} - |\Delta| a_i a_{i+1} + \text{h.c.}\right] - \mu \sum_{i=1}^{L}\left(a_i^\dagger a_i - \frac{1}{2}\right)$$

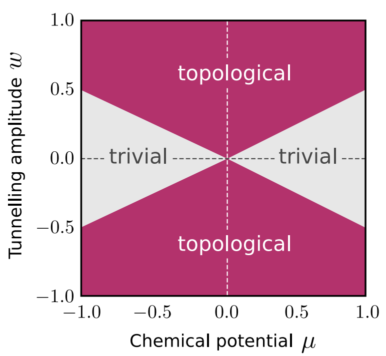


$$c_{2i-1} \equiv a_i + a_i^\dagger \quad \text{and} \quad c_{2i} \equiv i\left(a_i^\dagger - a_i\right) \quad \text{for} \quad i = 1, \ldots, L$$

Self-adjoint fermions
(Majorana fermions)

$$\{c_l, c_m\} = 2\delta_{l,m} \quad \text{and} \quad c_l = c_l^\dagger$$

The Majorana Chain # Phases



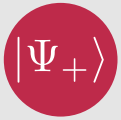
A Topological phase

Dominant hopping & pairing

$$H = iw \sum_{i=1}^{L-1} c_{2i} c_{2i+1}$$

$$\tilde{a}_i \equiv \frac{1}{2}(c_{2i} + i c_{2i+1})$$

Two degenerate ground states:

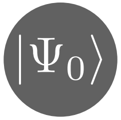


B Trivial phase

Dominant chemical potential

$$H = -\mu \frac{i}{2} \sum_{i=1}^{L} c_{2i-1} c_{2i}$$

Unique ground state:



Beyond Mean Field Models?

$$H = -\sum_{i=1}^{L-1}\left[w a_i^\dagger a_{i+1} - |\Delta| a_i a_{i+1} + \text{h.c.}\right] - \mu \sum_{i=1}^{L}\left(a_i^\dagger a_i - \frac{1}{2}\right)$$

No particle-number conservation
(due to mean field origin)

> Not justified in small, closed, number-conserving setups
> (e.g., optical lattices)

Particle-number conserving theory?

Previous Research

→ M. Cheng and H.-H. Tu (2011). Physical Review B, 84(9), 094503.
Majorana edge states in interacting two-chain ladders of fermions.

→ J. D. Sau et al. (2011). Physical Review B, 84(14), 144509.
Number conserving theory for topologically protected degeneracy in one-dimensional fermions.

→ L. Fidkowski et al. (2011). Physical Review B, 84(19), 195436.
Majorana zero modes in one-dimensional quantum wires without long-ranged superconducting order.

→ J. Ruhman et al. (2015). Physical Review Letters, 114, 100401.
Topological States in a One-Dimensional Fermi Gas with Attractive Interactions.

Bosonization

→ C. V. Kraus et al. (2013). Physical Review Letters, 111(17), 173004.
Majorana Edge States in Atomic Wires Coupled by Pair Hopping.

Numerical (DMRG)

→ G. Ortiz et al. (2014). Physical Review Letters, 113, 267002.
Many-body characterization of topological superconductivity: The Richardson-Gaudin-Kitaev chain.

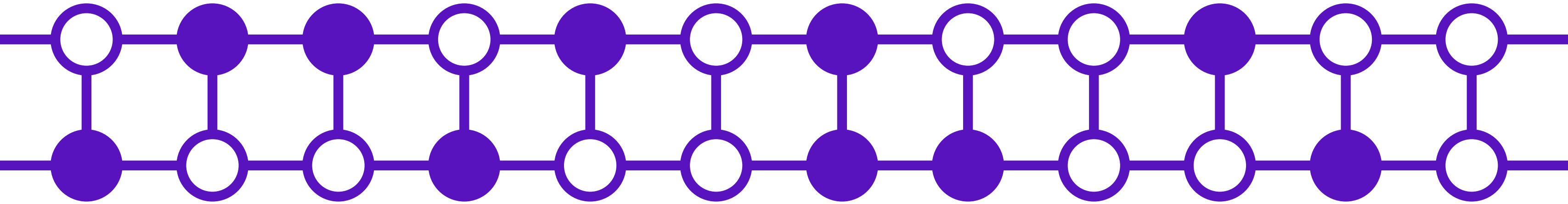
Long-Range

Short-range interacting theory
Exact ground state(s)
Majorana-like modes on edge

NL and HP Büchler, Phys. Rev. B 92, 041118(R) (2015)
Iemini et. al. , Phys. Rev. Lett. 115, 156402 (2015)

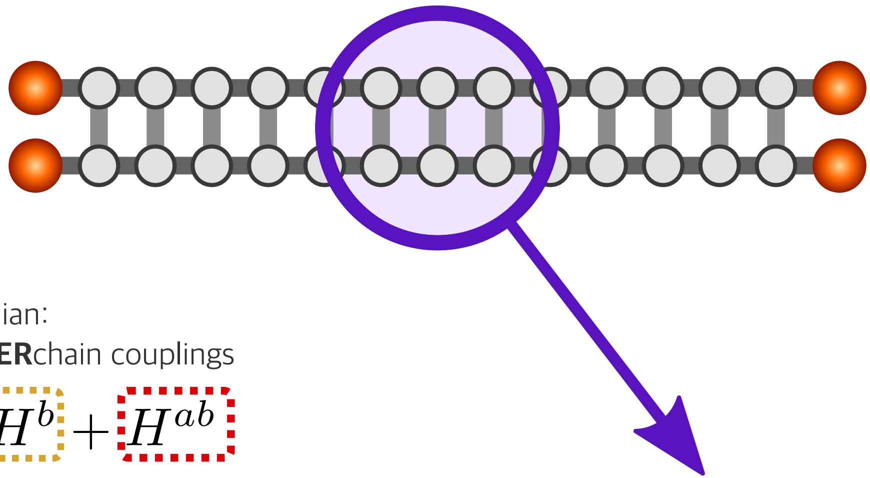
The Model

A Number-conserving Theory



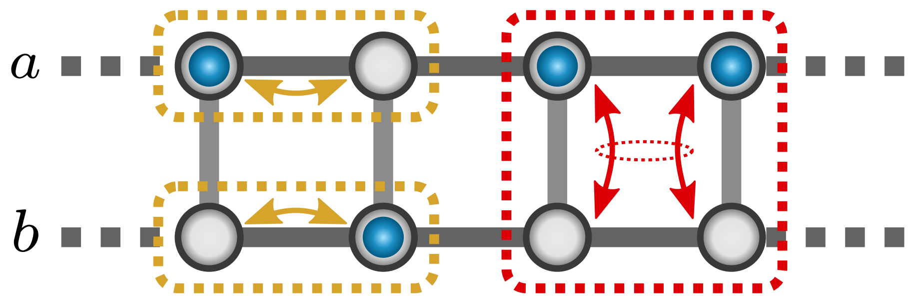
Setting

Open **DOUBLE** chain of spinless fermions ("ladder"):



Interacting Hamiltonian:
INTRAchain and **INTER**chain couplings

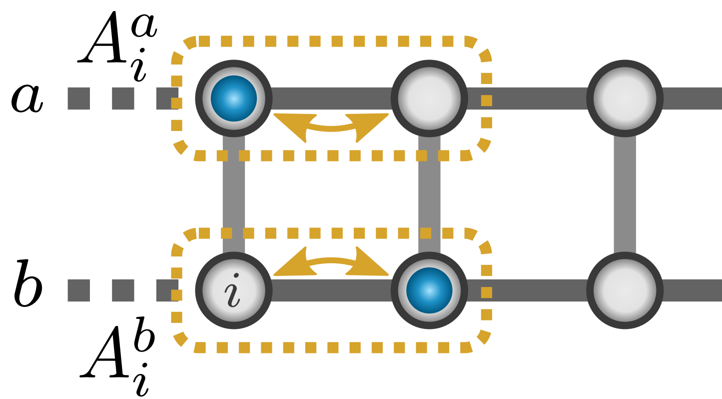
$$H = H^{a} + H^{b} + H^{ab}$$



Intrachain Physics

$$H^a = \sum_{i=1}^{L-1} A_i^a \left(\mathbb{1} + A_i^a\right)$$

$$a_i a_{i+1}^\dagger + a_{i+1} a_i^\dagger$$



- Locally positive-semidefinite operators

Single particle hopping + density-density interactions :

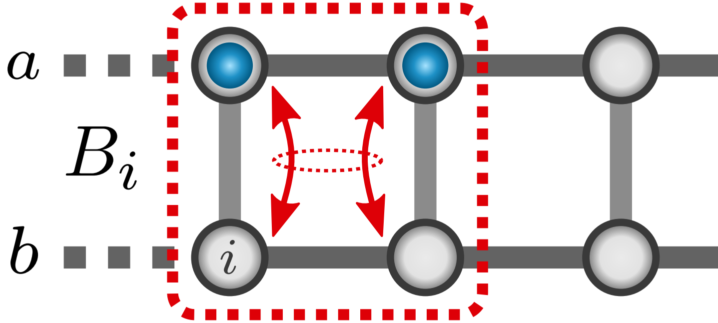
$$H_i^a = -a_i^\dagger a_{i+1} - a_{i+1}^\dagger a_i + n_i^a(1 - n_{i+1}^a) + n_{i+1}^a(1 - n_i^a)$$

Interchain Physics

$$H^{ab} = \sum_{i=1}^{L-1} B_i \left(\mathbb{1} + B_i\right)$$

$$a_i^\dagger a_{i+1}^\dagger b_i b_{i+1} + b_i^\dagger b_{i+1}^\dagger a_i a_{i+1}$$

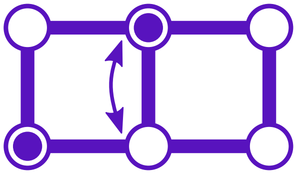
- Locally positive-semidefinite operators



Pair hopping + Pair density-density interactions:

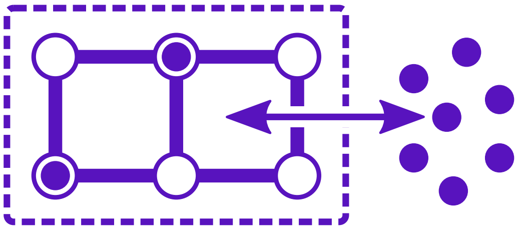
$$H_i^{ab} = -a_i^\dagger a_{i+1}^\dagger b_{i+1} b_i - b_i^\dagger b_{i+1}^\dagger a_{i+1} a_i$$
$$+ n_i^a n_{i+1}^a (1 - n_i^b)(1 - n_{i+1}^b) + n_i^b n_{i+1}^b (1 - n_i^a)(1 - n_{i+1}^a)$$

Symmetries



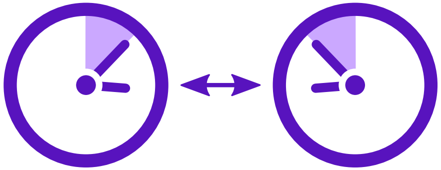
Subchain Parity

$$P_a = \prod_i (-1)^{n_i^a}$$



Total Particle Number

$$N = \sum_i \left(n_i^a + n_i^b\right)$$



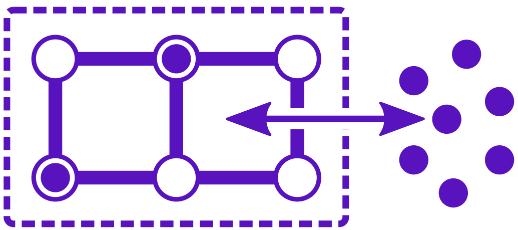
Time Reversal

$$\mathcal{T} = (\,\bullet\,)^*$$

Symmetries

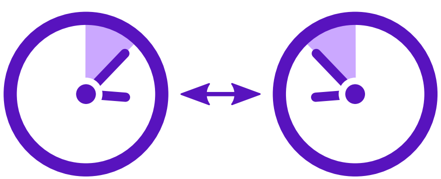
Subchain Parity

$$P_a = \prod_i (-1)^{n_i^a}$$



Total Particle Number

$$N = \sum_i \left(n_i^a + n_i^b\right)$$



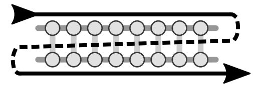
Time Reversal

$$\mathcal{T} = (\,\bullet\,)^*$$

Results

Exact Ground States & Braiding

Exact Ground States

1. Choose gauge (Fock basis) conveniently: 
2. Exploit local positivity of Hamiltonian (frustration free)
3. Construct ground states rigorously from scratch ...
4. Two degenerate zero-energy ground states in each N sector:

Equal-weight superposition of all configurations with fixed total particle number & subchain parity.

Total parity

$P = +1$

even	odd
even	odd

Subchain parity α

$P = -1$

even	odd
odd	even

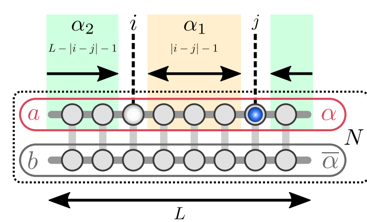
$$|N, \alpha\rangle = \mathcal{N}_{L,N,\alpha}^{-1/2} \sum_{n,(-1)^n=\alpha} |n\rangle_a |N-n\rangle_b$$

$$|n\rangle_x \equiv \sum_{|\boldsymbol{n}|=n} |\boldsymbol{n}\rangle_x$$

Evaluate all correlators & expectation values efficiently using combinatorics:

$$\langle N, \alpha| a_i^\dagger a_j |N, \alpha\rangle = \binom{L, L}{\alpha}_N^{-1} [\Lambda_{+1,-\alpha} - \Lambda_{-1,\alpha}]$$

Generalised Binomial Coefficients



Exact Ground States # Comparison

N-conserving

- Two degenerate ground states in each N-sector:

$P = +1$			$P = -1$	
even	odd	or	even	odd
even	odd		odd	even

- Zero-energy ground states:

Equal-weight superposition of all configurations with fixed total particle number & subchain parity.

Majorana Chain $(w = \Delta,\, \mu = 0)$

- Two degenerate ground states:

$P = +1$	$P = -1$
even	odd

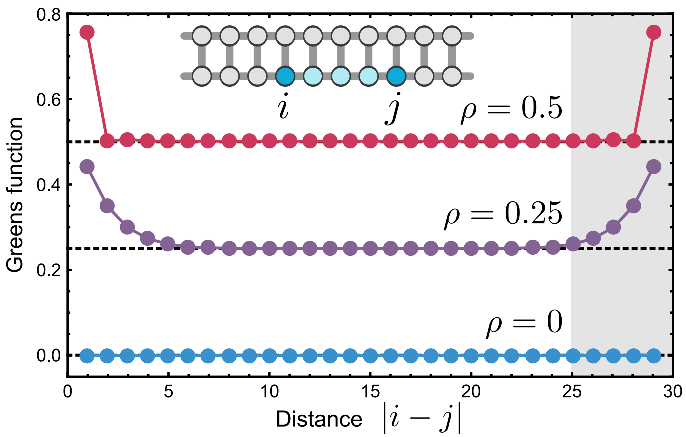
- Zero-energy ground states:

Equal-weight superposition of all configurations with fixed total parity.

Topological Properties # Edge States

INTRA-chain single-particle correlation

$\langle a_i^\dagger a_j \rangle$

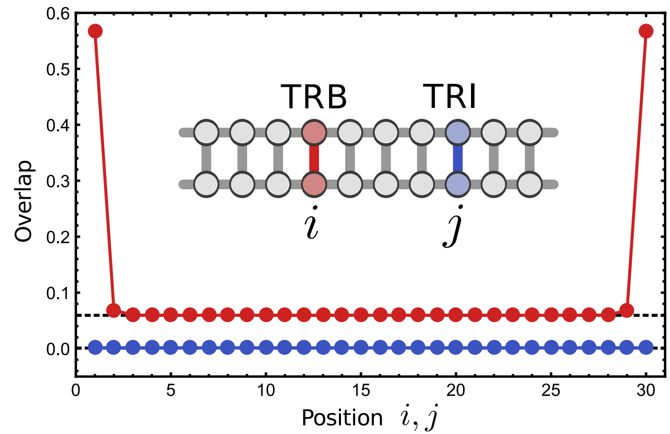


Exponential decay away from edges ("Edge states")

INTER-chain single-particle hopping overlap

TRI : $\langle -\alpha | \, a_\delta^\dagger b_\delta + b_\delta^\dagger a_\delta \, | \alpha \rangle \to 0$

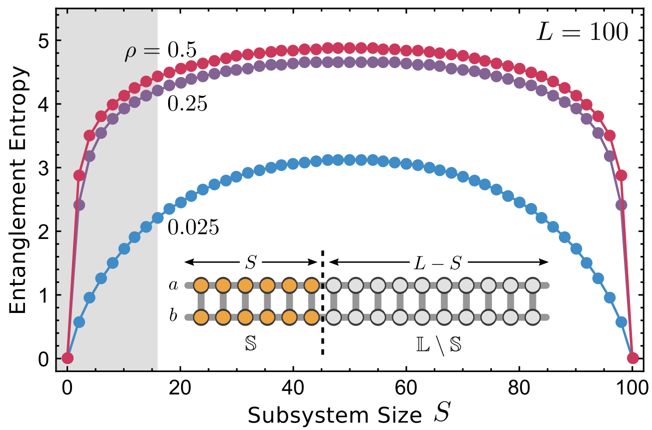
TRB : $\langle -\alpha | \, i a_\delta^\dagger b_\delta - i b_\delta^\dagger a_\delta \, | \alpha \rangle \to e^{-\mu(\rho)\delta}$



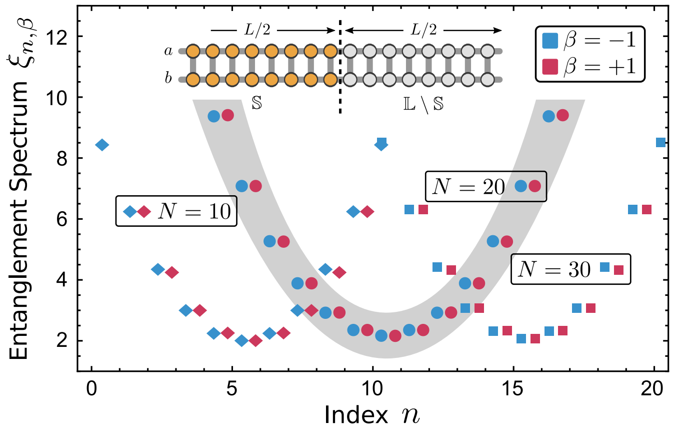
Only time-reversal breaking (TRB) hopping lifts degeneracy at the edges

Topological Properties # Entanglement

Entanglement Entropy



Entanglement Spectrum



- Entanglement entropy for growing subsystem
- Logarithmic corrections to area law
  → gapless spectrum!

- Entanglement spectrum for half-splitted double chain
- Stable two-fold degeneracy in thermodynamic limit

Excitations & Low-energy Physics

Single-particle excitations:

Expectation:
Number-conservation → Gapless Goldstone modes

Symmetric superposition of single-magnon states:

$$|k; N, \alpha\rangle = P_1^a(k) \oplus P_1^b(k)\ |N, \alpha\rangle$$

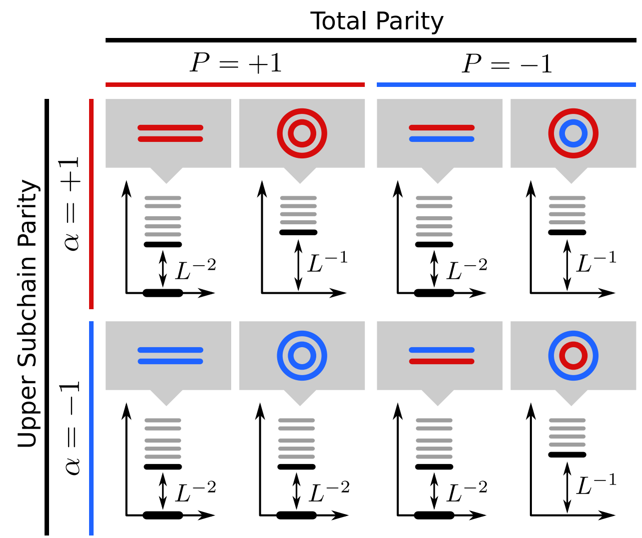
$$P_1^x(k) = \sum_{j=1}^{L} \cos\left[\tfrac{k}{2}(2j-1)\right](-1)^{x_j^\dagger x_j}$$

Quadratic, gapless spectrum:

$$E_k = 4\sin^2\tfrac{k}{2}$$

$$k = m\tfrac{\pi}{L},\ 0 \le m < L$$

Algebraically closing gap: L^{-2}



Braiding # Procedure

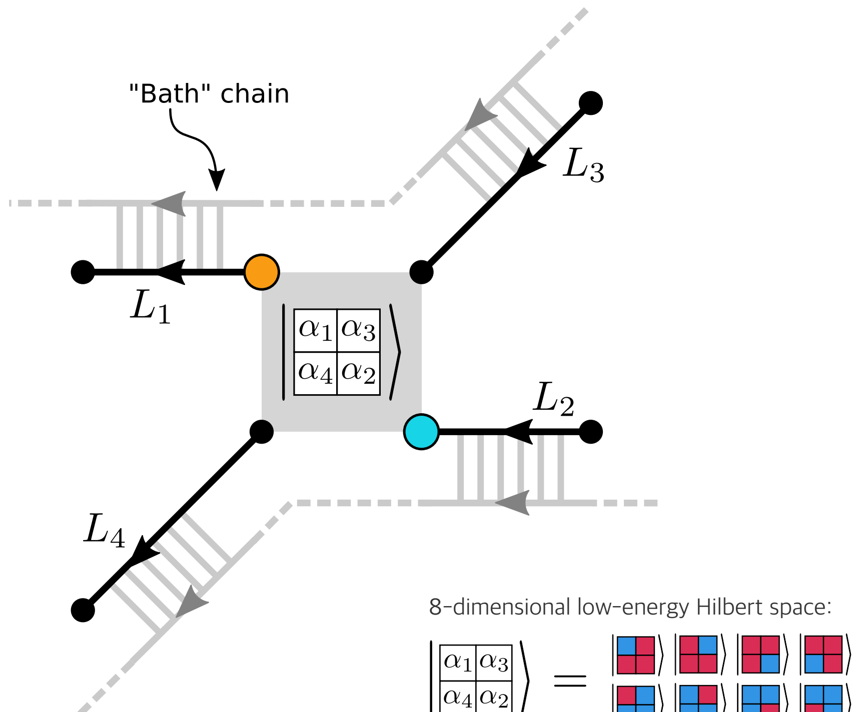
- Braid two localized edge states around each other:

- Consider 4 weakly coupled chains L_i paired with a common "bath" chain

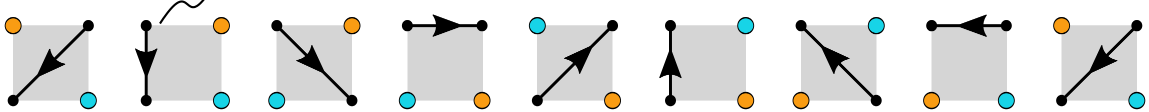
- Vary weak single-particle couplings adiabatically:

$$H(t) = H_0 + \varepsilon/L^2\, H_{\text{int}}(t)$$

High-energy Hamiltonian (uncoupled wires)



Low-energy single-particle coupling

$H_{\text{int}}(t) =$ 

Braiding # Non-Abelian Statistics

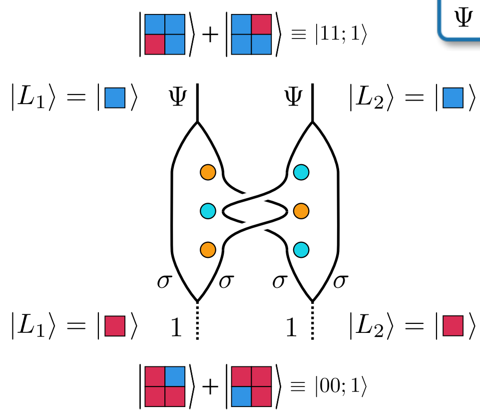
- Numerical evaluation of adiabatic time evolution:

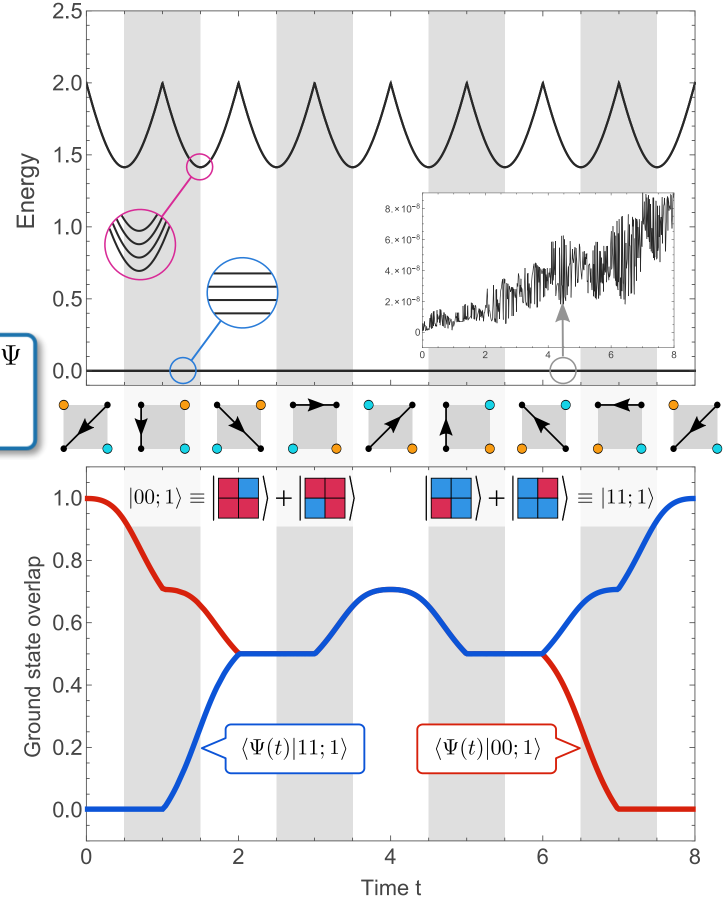
  - Non-Abelian Holonomy:

$$|11;1\rangle \;=\; \exp\left[-i\int \mathrm{d}t\, H_{\text{int}}(t)\right]|00;1\rangle$$

  - Compare with Ising TQFT: $\{1, \sigma, \Psi\}$ ("Majorana" = σ, "Fermion" = Ψ)

$$\sigma\otimes\sigma = 1\oplus\Psi$$
$$\sigma\otimes\Psi = \sigma$$
$$\Psi\otimes\Psi = 1$$





- Edge states = Ising anyons

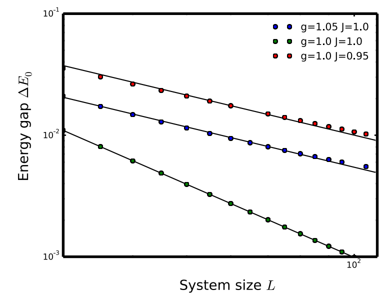
Outlook

DMRG & more

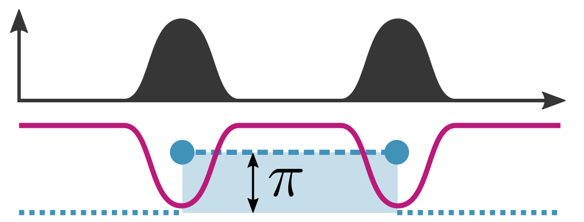
Current Research

Explore nearby phases by
MPS based DMRG simulations

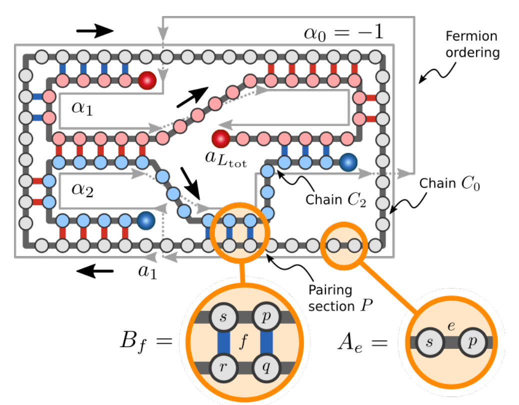
Kai Guther, Masterthesis, ITP III



Explore low-energy sector by ansatz wave functions
(using the brain, not DMRG)



Explore more complex ladder networks
for braiding procedures



Questions?

Poster