Exploring quantum phases by driven dissipation

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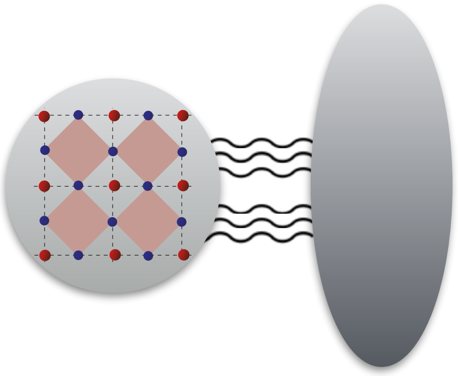
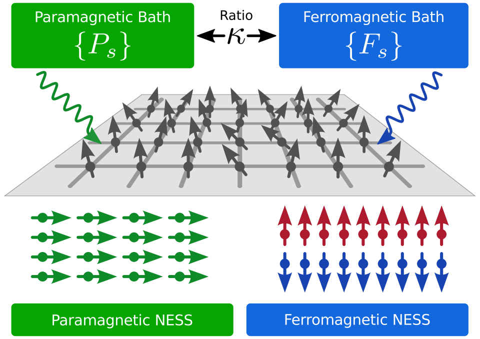
SFB TRR21
Tailored quantum matter

Outline



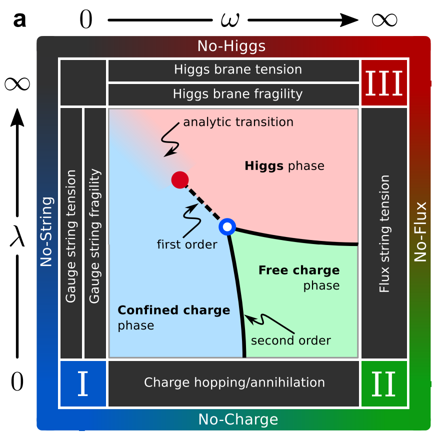
1. General concept

Lindblad master equation



2. Dissipative quantum phase transitions

Paradigmatic model of a purely dissipative system



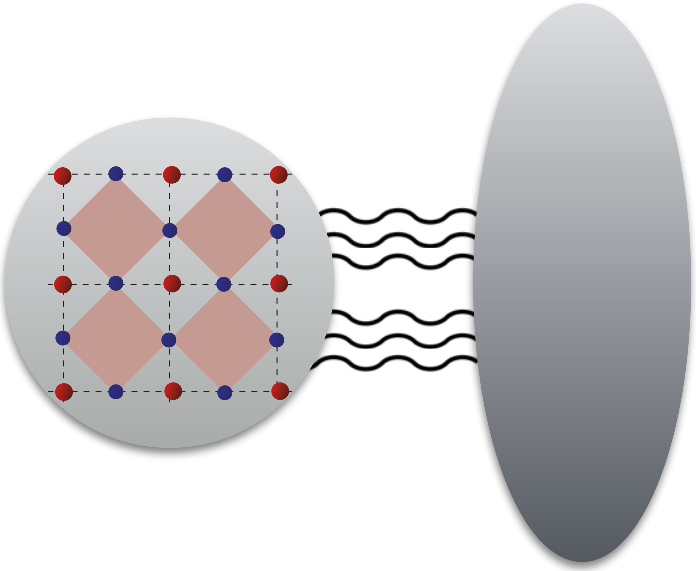
3. Lattice gauge theory

Dissipative implementation

1

General concept

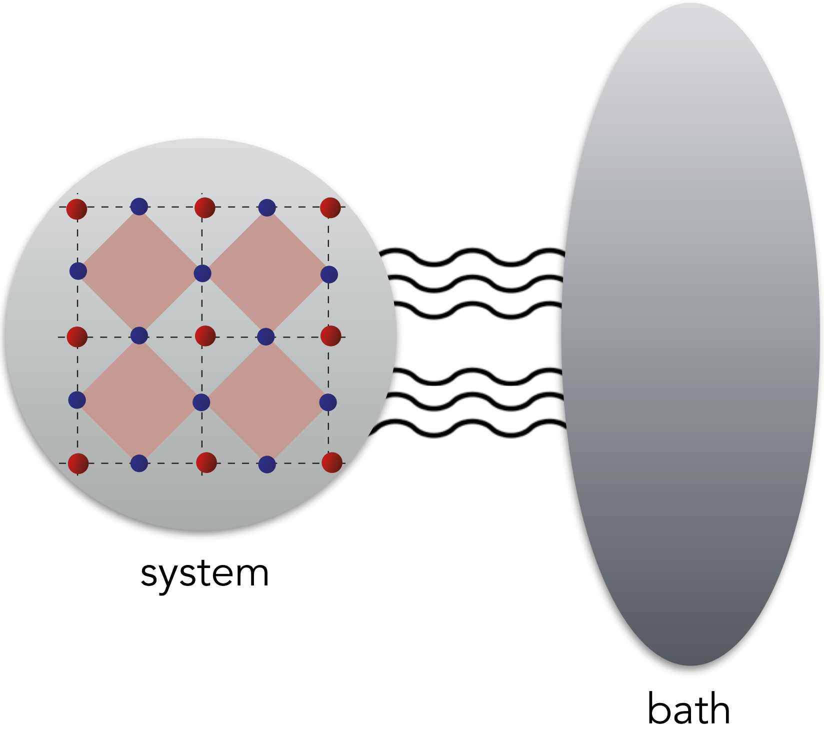
Lindblad master equation



Dissipation and decoherence

Master equation

- coupling between system and reservoirs
- dephasing and decoherence
- Born-Markov approximation
  - - no-memory of the reservoir
  - - weak coupling between system and bath



Lindblad master equation

$$\partial_t \rho = \mathcal{L}\left[\rho\right] = \sum_{\alpha} \kappa_\alpha \left[c_\alpha \rho c_\alpha^\dagger - \frac{1}{2} c_\alpha^\dagger c_\alpha \rho - \frac{1}{2} \rho c_\alpha^\dagger c_\alpha \right]$$

c_α : jump operator

- e.g., optical master equation
  laser cooling

Dark states

Dark states

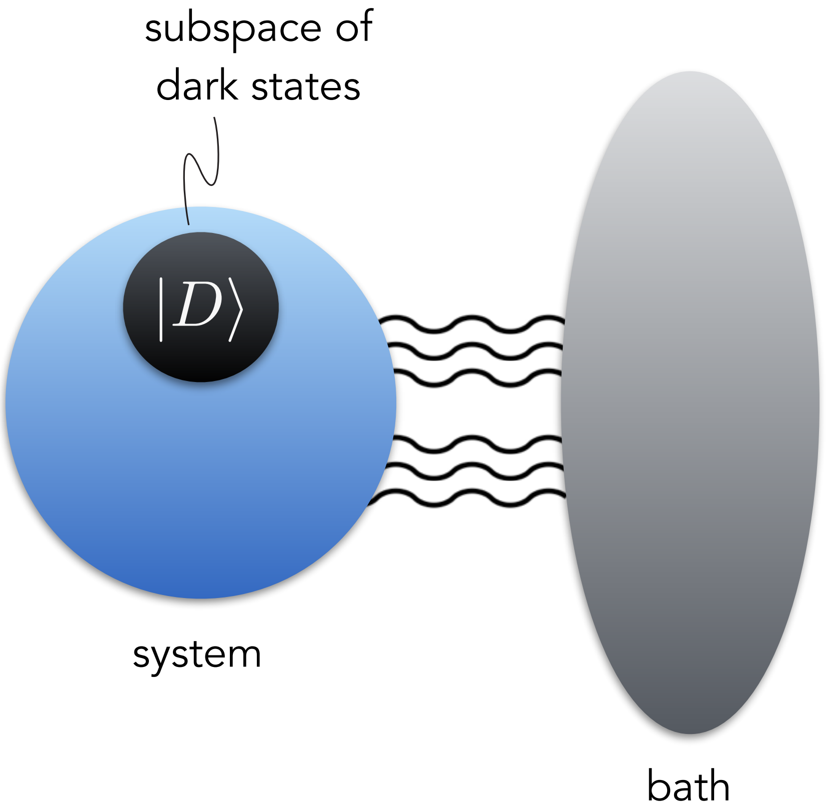
- eigenstates of all jump operators with vanishing eigenvalue

$$c_\alpha |D\rangle = 0$$

- pure state

$$\rho = |D\rangle\langle D|$$

- decoherence free subspace
- stationary solution of the master equation



$$\partial_t \rho = \mathcal{L}[\rho] = \sum_\alpha \kappa_\alpha \left[c_\alpha |D\rangle\langle D| c_\alpha^\dagger - \frac{1}{2} c_\alpha^\dagger c_\alpha |D\rangle\langle D| - \frac{1}{2} |D\rangle\langle D| c_\alpha^\dagger c_\alpha \right] = 0$$

Goal:

- engineering of jump operator with desired state a dark state
- dark state is unique stationary solution

Dephasing versus cooling

Dephasing

- hermitian jump operator

$$c_\alpha^\dagger = c_\alpha$$

- each eigenstate is stationary state
- diagonal density matrix

$$c_\alpha^\dagger|\lambda\rangle = \lambda|\lambda\rangle$$

$$\rho = \sum_{\lambda,\mu} c_{\lambda\mu}|\lambda\rangle\langle\mu| \rightarrow \sum_{\lambda} p_\lambda|\lambda\rangle\langle\lambda|$$

loss of coherence

Cooling

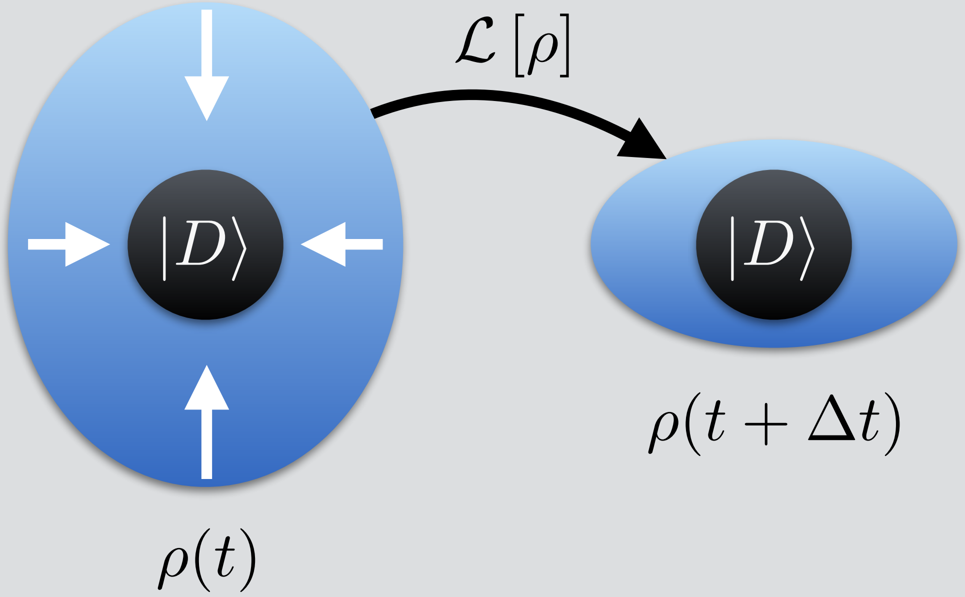
- non-hermitian jump operator

$$c_\alpha^\dagger \neq c_\alpha$$

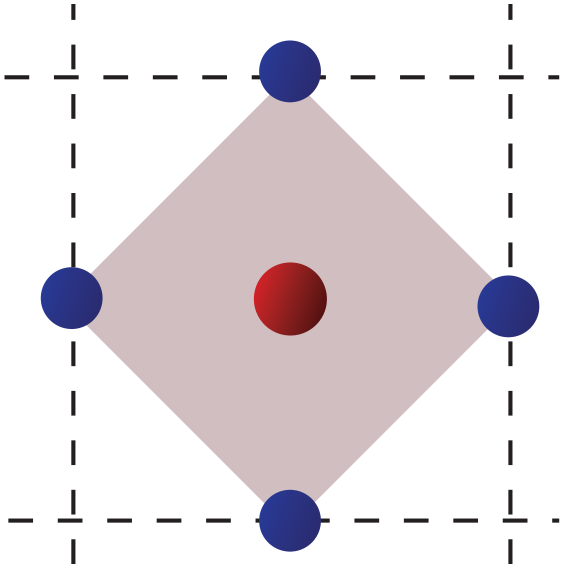
- preparation into the subspace of dark states

- arbitrary initial density matrix evolves into unique pure state

$$\rho \rightarrow |D\rangle\langle D|$$



Implementation



Digital quantum simulation

- Implementation with Rydberg atoms
  H. Weimer, et al., Nature Physics 6, 382 (2010)
- Implementation with Ion traps
  Barreiro, et al., Nature 470, 486 (2011)

Dissipative element:
spontaneous emission, optical pumping

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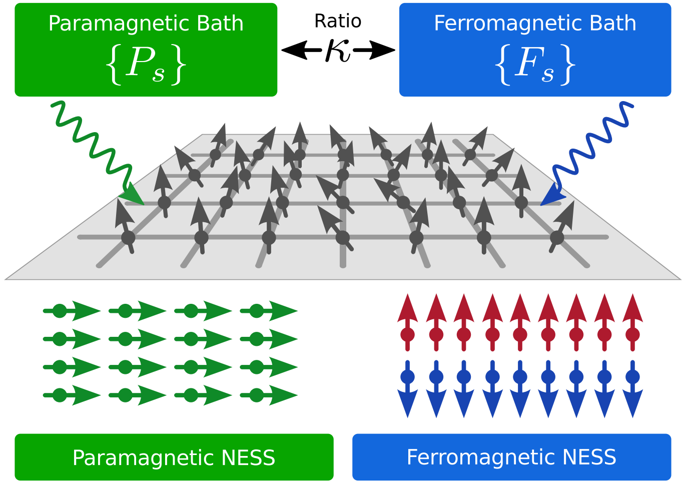
Dissipative quantum phase transitions

Paradigmatic model of a purely dissipative system



Example: Paramagnet

Spin system in dimension d

- spins at lattice sites s

$$P_s = \sqrt{\kappa}\,\underbrace{\sigma_s^z}_{\text{THEN}}\,\underbrace{[1-\sigma_s^x]}_{\text{IF}}$$

- unique dark state

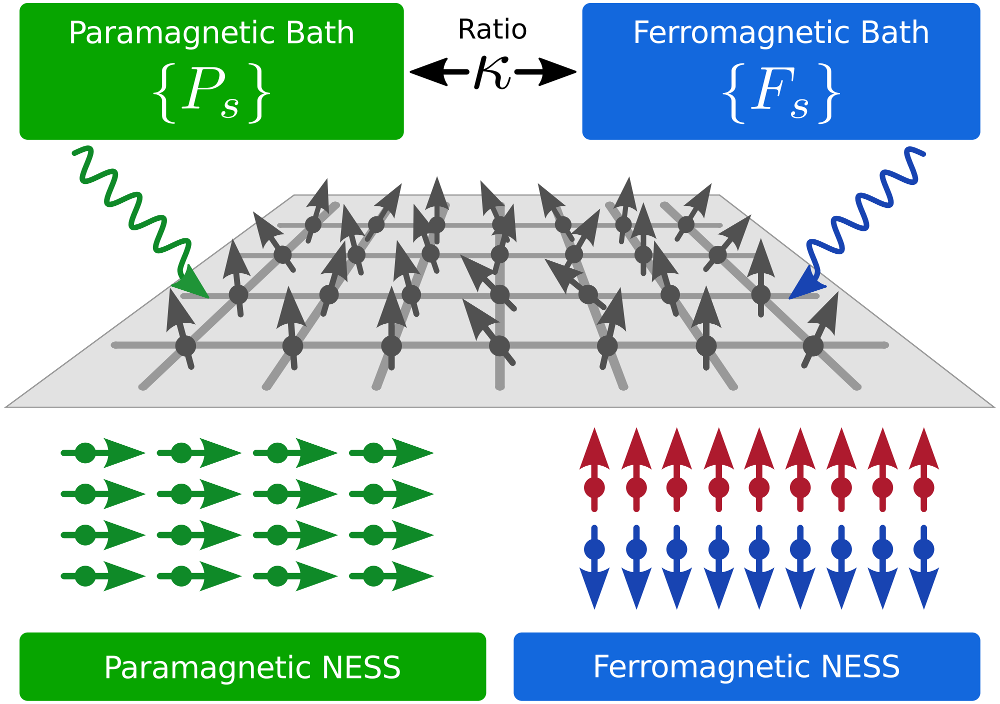
$$|D\rangle = \prod_s |\rightarrow\rangle_s$$

- parent Hamiltonian

$$H = \sum_s P_s^\dagger P_s$$

$$= 2\kappa \sum_s [1-\sigma_s^x]$$

: frustration free,
unique zero energy ground state
external magnetic field along x-direction



Paramagnetic Bath
$\{P_s\}$

Ratio
$\leftarrow\kappa\rightarrow$

Ferromagnetic Bath
$\{F_s\}$

Paramagnetic NESS

Ferromagnetic NESS

Example: Ferromagnet

Spin system in dimension d

- spins at lattice sites s

number of nearest neighbors

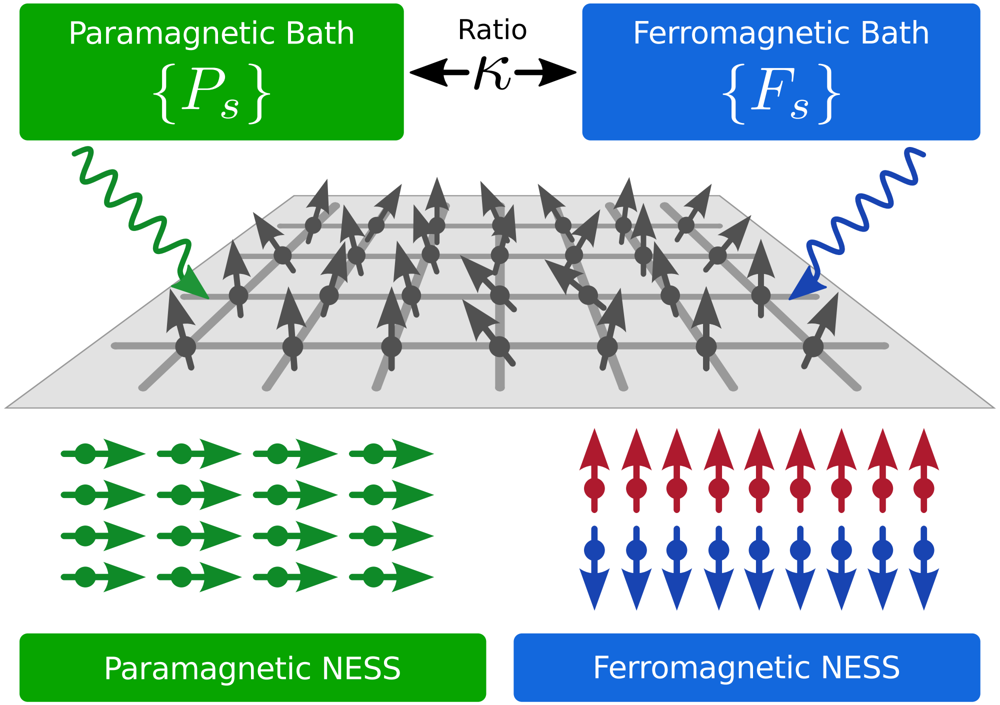
$$F_s = \underbrace{\sigma_s^x}_{\text{THEN}} \underbrace{\left[1 - \frac{1}{q}\sum_{t\in s}\sigma_t^z\sigma_s^z\right]}_{\text{IF}}$$

- two dark states:
  two ferromagnetic states

$$|D\rangle = \prod_s |\uparrow\rangle_s$$

$$|D\rangle = \prod_s |\downarrow\rangle_s$$

- parent Hamiltonian:
  ferromagnetic Ising model



Paramagnetic Bath $\{P_s\}$

Ratio κ

Ferromagnetic Bath $\{F_s\}$

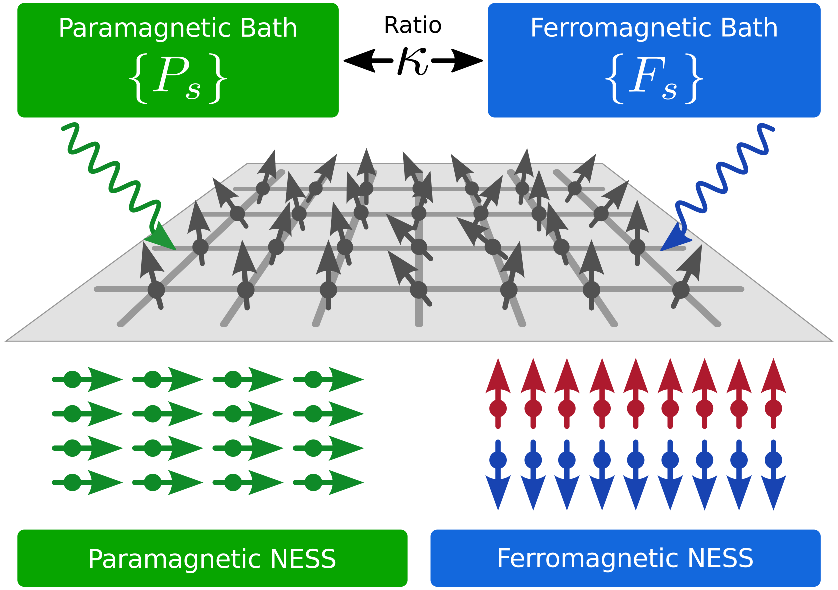
Paramagnetic NESS

Ferromagnetic NESS

Exploring quantum phases

Non-equilibrium steady state phase diagram?

- both competing drives
- is there a phase transition?
- does the phase diagram resemble the “blue-print” Hamiltonian system?



Paramagnetic Bath

$\{P_s\}$

Ratio

$\longleftarrow \kappa \longrightarrow$

Ferromagnetic Bath

$\{F_s\}$

Paramagnetic NESS

Ferromagnetic NESS

$$\partial_t \rho = \sum_s \left[P_s \rho P_s^\dagger - \frac{1}{2} P_s^\dagger P_s \rho - \frac{1}{2} \rho P_s^\dagger P_s \right] + \left[F_s \rho F_s^\dagger - \frac{1}{2} F_s^\dagger F_s \rho - \frac{1}{2} \rho F_s^\dagger F_s \right]$$

- parent Hamiltonian: transverse Ising model

$$H = - \sum_{\langle s,t \rangle} \sigma_s^z \sigma_t^z - \kappa \sum_s \sigma_s^x$$

Coherent and dissipative dynamics

Phase transitions and metastability for competing dissipative and coherent drives

Bose-Hubbard, Rydberg atoms,
Fermionic systems, conceptional questions

- Diehl, Zoller, Fazio PRL (2010)
- Lee, Cross (2011)
- Lesanovsky,
- Maria Ray, Hazzard (2013)
- Eisert (2012)
- Fleischhauer, Moos, Höning (2012)
- Shirai, Mori, Miyashita (2014)
- Immamoglu, Cirac, Lukin (2012)

Here:

Only dissipative dynamics with quantum mechanics encoded in non-commuting jump operators

cf. several talks in WP4 yesterday

Exploring quantum phases

Methods

- Wave function Monte Carlo simulation of master equation: only small systems
- DMRG simulations: only in one dimension
- Keldysh path integral formulation

Mean-field theory

- exact in high dimensions
- ansatz for density matrix:

$$\rho = \prod_s \rho_s$$

- homogeneous density matrix

$$\rho_s \equiv \hat{\rho}(\mathbf{m})$$

partial trace

⇨

- effective master equation for

$$\partial_t \hat{\rho}(\mathbf{m}) = \mathcal{L} \hat{\rho}(\mathbf{m})$$

- self-consistency

$$m_\alpha = \mathrm{Tr}\left[\sigma^\alpha \hat{\rho}(\mathbf{m})\right]$$

Mean-field theory

Paramagnetic jump operators

- local on each lattice site and remain the same within mean-field theory

$$f_0 = \sqrt{\kappa}\sigma^z\left[1 - \sigma^x\right]$$

Ferromagnetic jump operators

- ferromagnetic drive

$$f_1 = \sigma^x\left[1 - m_z\sigma^z\right]$$

- dephasing terms

$$f_2 = \frac{1}{\sqrt{2d}}\sqrt{1 - m_z^2}\sigma^y$$

$$f_3 = \frac{1}{\sqrt{2d}}\sigma^z$$

$$\partial_t\hat{\rho} = \sum_{i=0}^{3}\left[2f_i\hat{\rho}f_i^\dagger - f_i^\dagger f_i\hat{\rho} - \hat{\rho}f_i^\dagger f_i\right]$$

$$m_\alpha = \mathrm{Tr}\left[\sigma^\alpha\hat{\rho}\right] \qquad \hat{\rho} = \frac{1 + \mathbf{m}\sigma}{2}$$

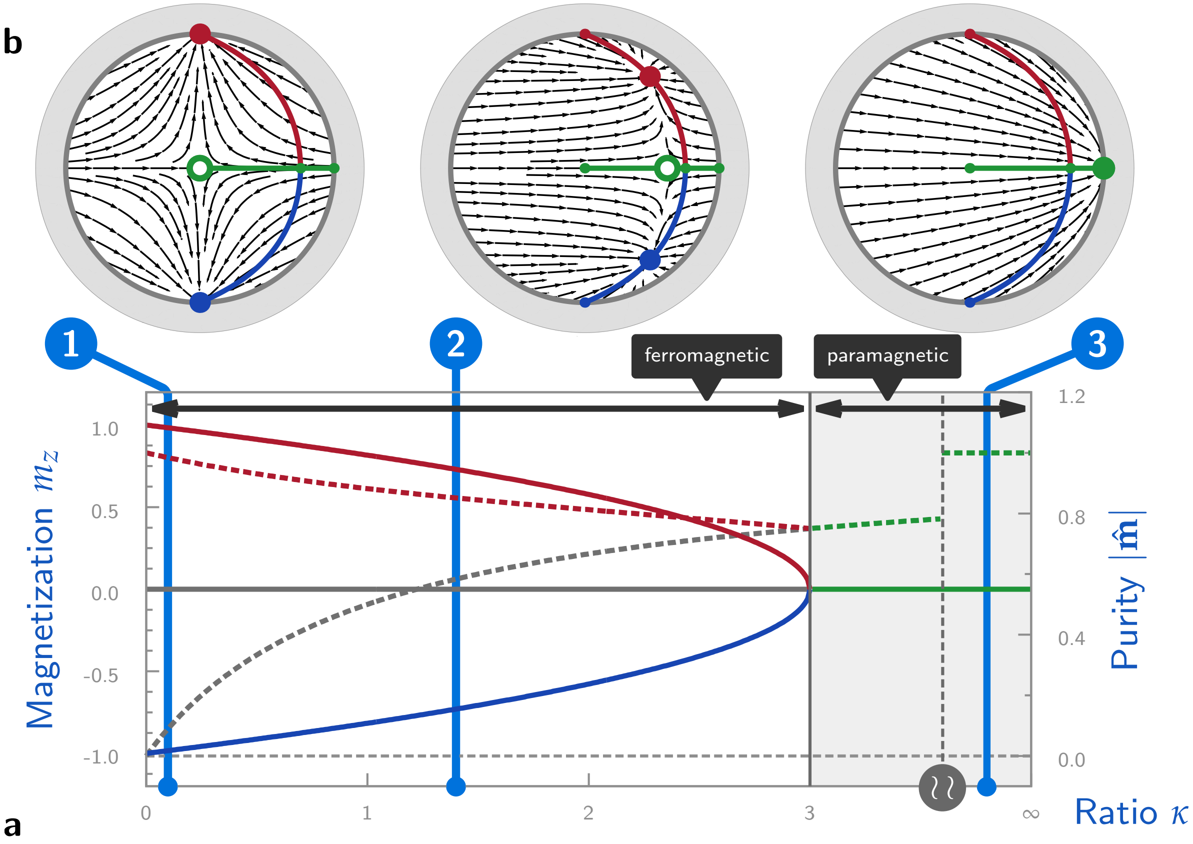
$$\Rightarrow \qquad \partial_t\mathbf{m} = \mathbf{F}(\mathbf{m})$$

three coupled
non-linear equations

Mean-field theory

Second order phase transition

- critical value: $\kappa_c = 4\left(1 - \frac{1}{2d}\right)$
- continuous behavior of the order parameter
- ferromagnetic to paramagnetic phase transition
- in general: mixed state, with finite purity
- purity is minimal at phase transition point
- behavior resembles the thermal phase diagram for the parent Hamiltonian
- critical exponents in analogy to mean-field exponents for the Hamiltonian system



Dissipative Transverse Ising model

Dissipative model

- second order phase transition
- critical value for drive;
  minimal purity at phase transition
- mean-field critical exponent
- gap in Lindblad spectrum with
  gapless point for critical drive

Hamiltonian system

- second order phase transition
- two-parameter phase diagram:
  temperature and transverse field
- identical critical exponents
- gapped system with gapless point
  at critical point

⇨ Can a dissipatively driven system explore the full richness of the Hamiltonian "blue print" model?

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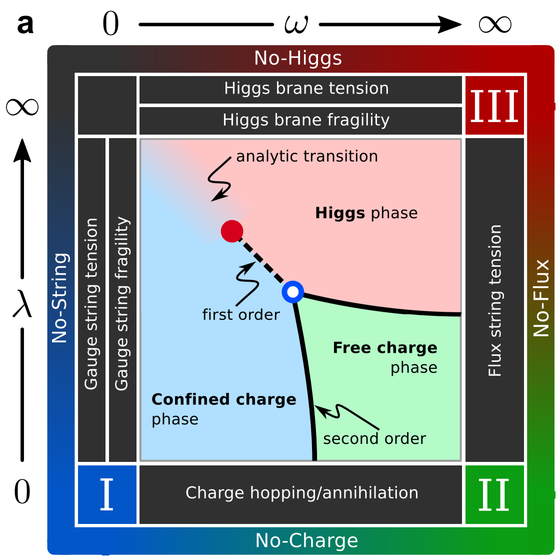
Lattice gauge theory

Dissipative implementation

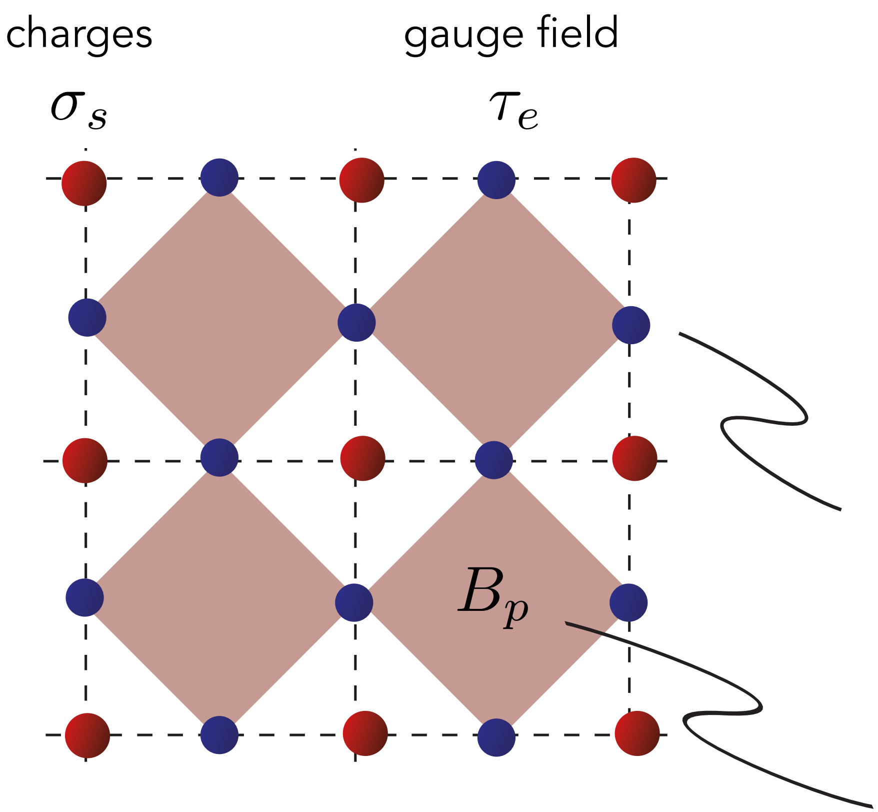


Lattice gauge theory

Z_2 lattice gauge Higgs model

- simplest model of gauge field and charged particles

$$H = -\underbrace{\sum_s \sigma_s^x}_{\text{chemical potential}} - \lambda \underbrace{\sum_e I_e}_{\text{kinetic energy}} - \underbrace{\sum_e \tau_e^x}_{\text{electric field}} - \omega \underbrace{\sum_p B_p}_{\text{magnetic field}}$$



Gauge symmetry:

$$[H, G_s] = 0 \qquad G_s \equiv \sigma_s^x \prod_{e \in s} \tau_e^x$$

$$I_e = \sigma_s^z \tau_e^z \sigma_{s'}^z$$: minimal coupling between matter and gauge field

$$B_p = \prod_{e \in p} \tau_e^z$$: magnetic flux

Wegner, F. J. Journal of Mathematical Physics 12, 2259 (1971)
Fradkin, Susskind, Physical Review D 17, 2637 (1978)
Fradkin, Shenker, Physical Review D 19, 3682 (1979)

Lattice gauge theory

Z₂ lattice gauge Higgs model

- simplest model of gauge field and charged particles

$$H = \underbrace{-\sum_s \sigma_s^x}_{\text{chemical potential}} \underbrace{- \lambda \sum_e I_e}_{\text{kinetic energy}} \underbrace{- \sum_e \tau_e^x}_{\text{electric field}} \underbrace{- \omega \sum_p B_p}_{\text{magnetic field}}$$

High energy approach

- all physical observable are gauge invariant

$$[A, G_s] = 0$$

⇨

physical states are equivalence classes of states in different gauges

e.g. Karl Jansen's talk yesterday

Condensed matter approach

- terms in the Hamiltonian enforce the gauge constraint

$$G_s|\psi\rangle = |\psi\rangle$$

⇨

emergent gauge theory at low energies

e.g. Alex Glätzle's talk yesterday

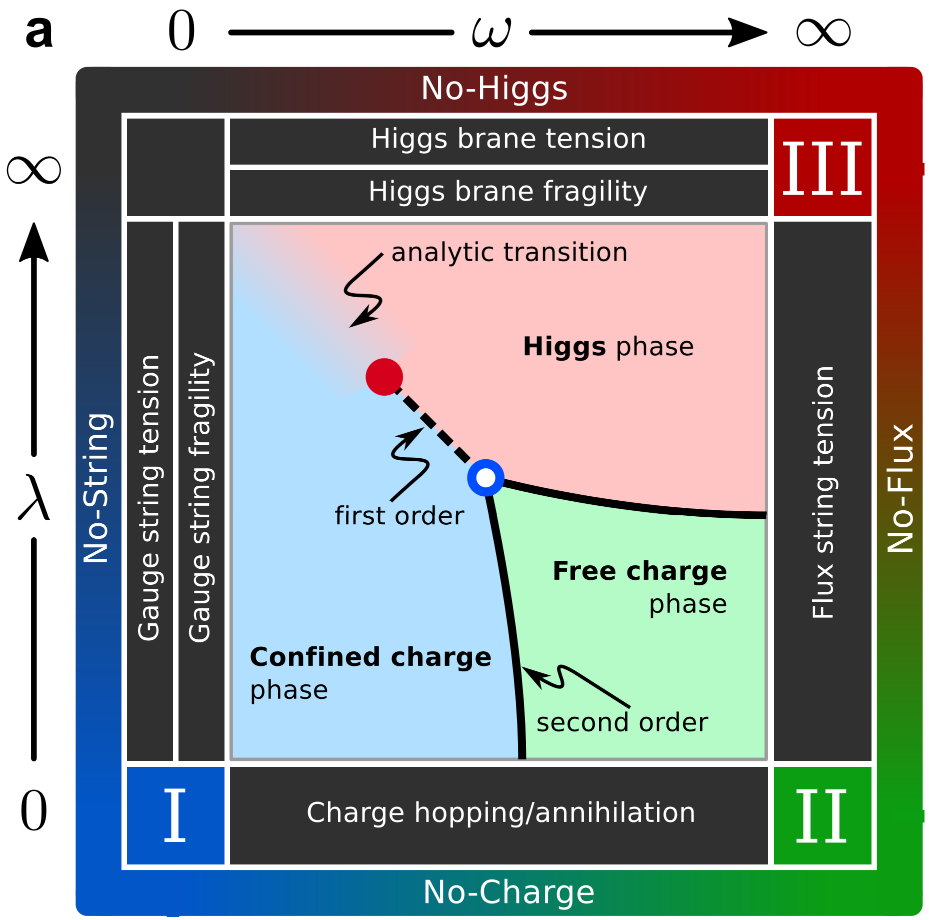
Lattice gauge theory

Z_2 lattice gauge Higgs model

- simplest model of gauge field and charged particles

$$H = -\sum_s \sigma_s^x - \lambda \sum_e I_e - \sum_e \tau_e^x - \omega \sum_p B_p$$

$$I_e = \sigma_s^z \tau_e^z \sigma_{s'}^z, \qquad B_p = \prod_{e \in p} \tau_e^z$$



Implementation of this model by dissipation?

- three corners of the phase diagram can be dissipatively prepared

Lattice gauge theory

Confining phase: $\lambda = \omega = 0$

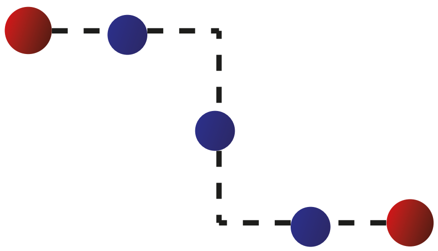
- Hamiltonian:

$$H = -\sum_s \sigma_s^x - \sum_e \tau_e^x$$

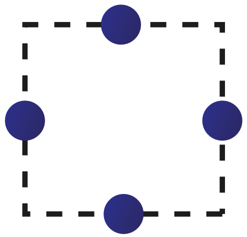
- gauge invariance:

$$G_s \equiv \sigma_s^x \prod_{e \in s} \tau_e^x$$

Fundamental excitations:



mesons



gauge loop

⇨ charges connected
by a string of electric field

Design jump operator to prepare into the ground state:

- "naive approach" breaks gauge invariance

$$\xcancel{\sigma_s^z \left[1 - \sigma_s^x\right] \quad \tau_e^z \left[1 - \tau_e^x\right]}$$

⇨ require gauge invariant
jump operators

Lattice gauge theory

Confining phase:

$\lambda = \omega = 0$

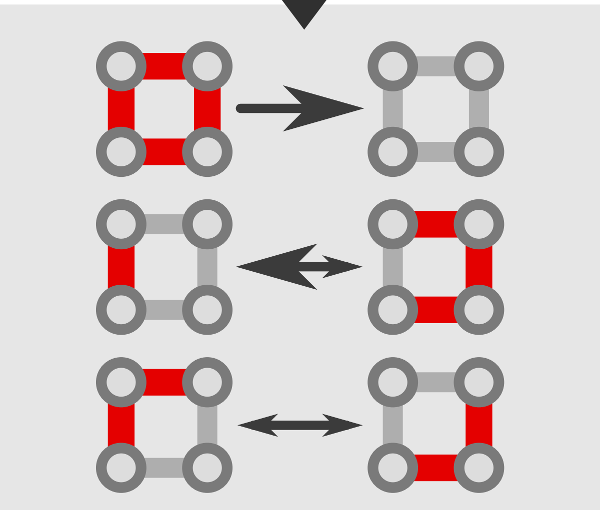
⇨

pure state as
steady state

Removing gauge loops
and deformation of loops:

$$B_p \left[1 - \frac{1}{q} \sum_{e \in p} \tau_e^x \right]$$

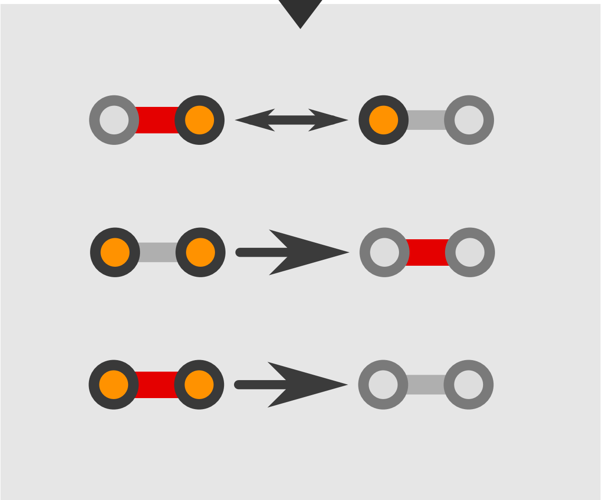
Gauge string tension



Removing confined charges
and hopping of charges:

$$I_e \left[1 - \frac{1}{2} \sum_{s \in e} \sigma_s^x \right]$$

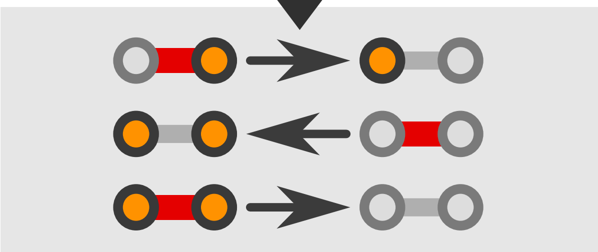
Charge hop./anihil.



Breaking of topological
gauge loops:

$$I_e \left[1 - \tau_e^x \right]$$

Gauge string fragility

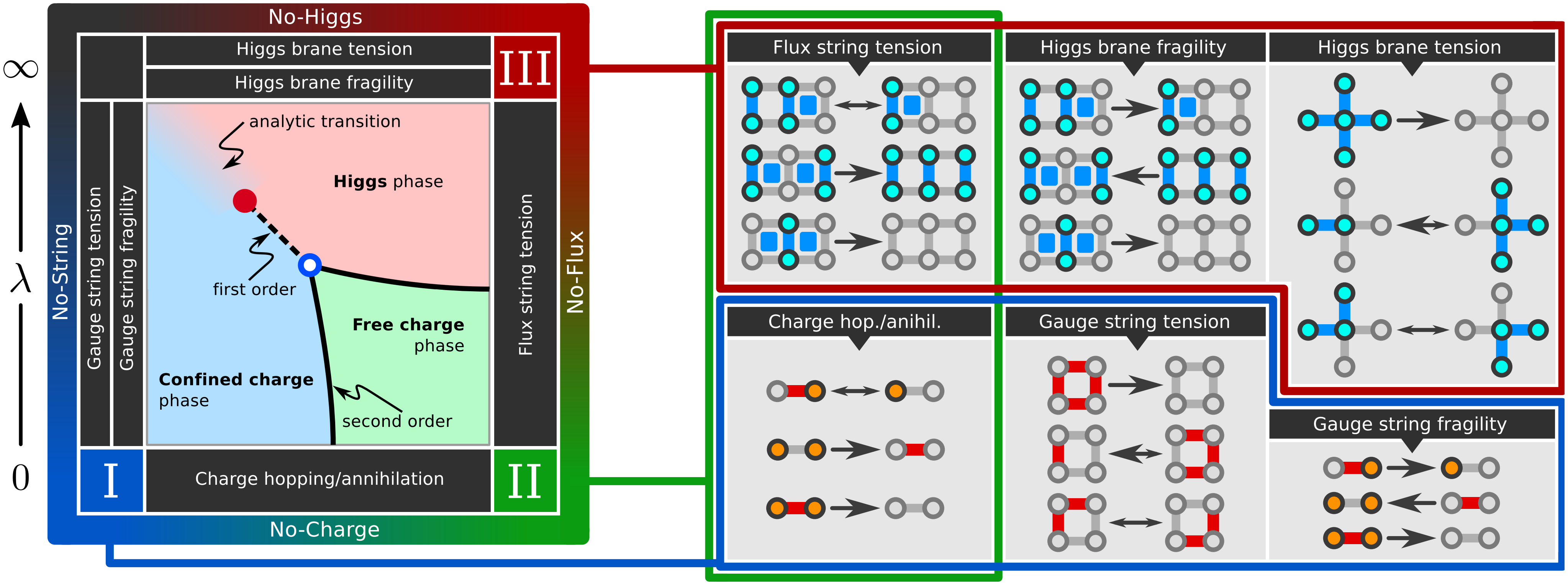


Lattice gauge theory

Full set of gauge invariant jump operators

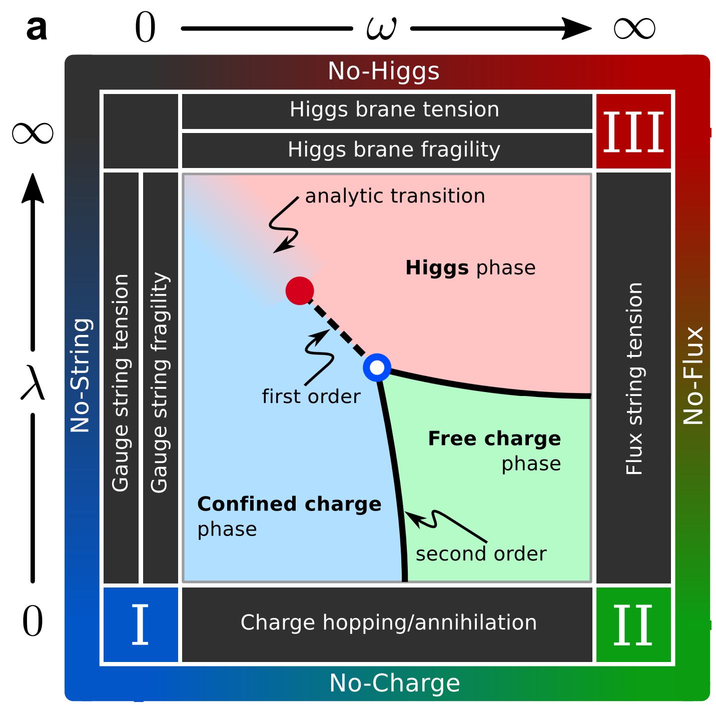
- requires 6 jump operators
- three edges of the phase diagram can be prepared

Bath	**Jump operator**
Gauge string tension	$F_p^{(1)} = \eta_1 \; B_p \left(\mathbb{1} - \tau^x_{e\in p}\right)$
Gauge string fragility	$F_e^{(2)} = \eta_2 \, I_e \left(\mathbb{1} - \tau^x_e\right)$
Higgs brane tension	$D_s^{(1)} = \eta_3 \, \sigma^x_s \left(\mathbb{1} - I_{e\in s}\right)$
Higgs brane fragility	$D_e^{(2)} = \eta_4 \, \tau^x_e \left(\mathbb{1} - I_e\right)$
Charge hopping	$T_e = \eta_5 \, I_e \left(\mathbb{1} - \sigma^x_{s\in e}\right)$
Flux string tension	$B_e = \eta_6 \, \tau^x_e \left(\mathbb{1} - B_{p\in e}\right)$



Lattice gauge theory

Exploring the full phase diagram by competing dissipative drives



Mean-field theory for lattice gauge model

- two mean-fields:

$$\rho = \prod_e \rho_e \prod_s \rho_s$$

- all three phases are predicted within mean-field theory
- well known artifacts of MF for lattice gauge theories

⇨

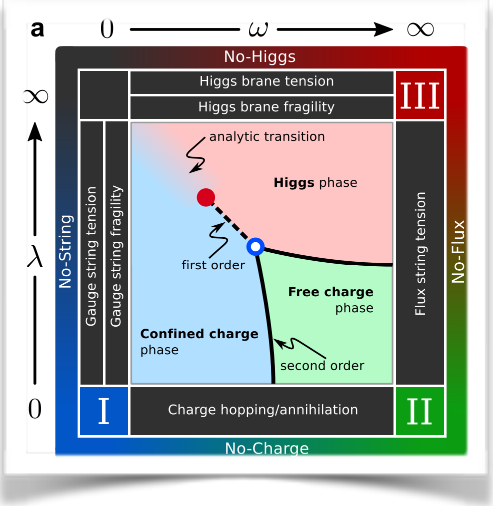
Dissipative MF phase diagram parallels the well-known MF phase diagram of the Hamiltonian theory

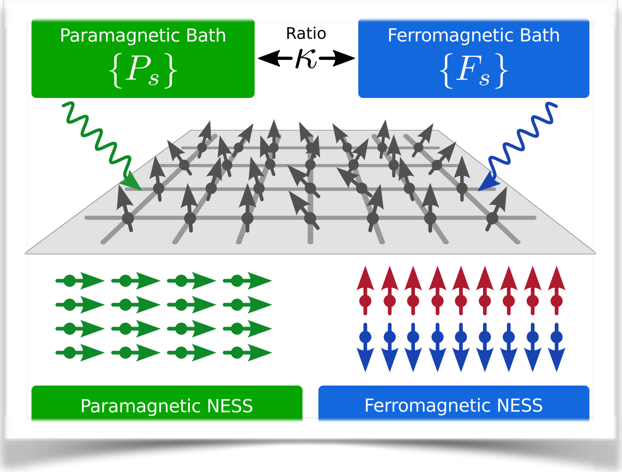
Drouffe, Zuber, Phys. Rep. 102,1 (1983)

Conclusion

Exploring quantum phases by driven dissipation

- paradigmatic transverse Ising model
- reveals the Hamiltonian phase diagram in high dimensions





Lattice gauge theory

- first demonstration how to implement a lattice gauge theory by dissipation
- resembles the phase diagram of the Hamiltonian system

Do dissipative systems in general reveal the different ground state phases of the “blue-print” Hamiltonian system?