Entanglement in Quantum Many-Body Systems

In and Out of Equilibrium

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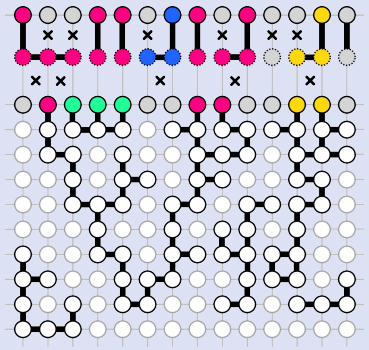
Institute for Theoretical Physics

Overview

Part I

Entanglement phase transitions in monitored quantum systems

Can entanglement survive projective measurements?

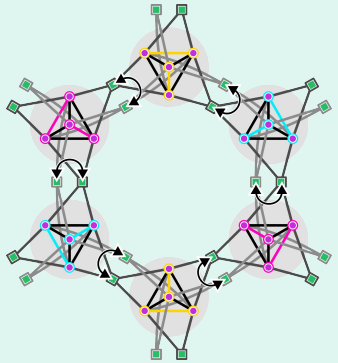


- **Entanglement Transition in the Projective Transverse Field Ising Model**
  NL and Hans Peter Büchler
  Phys. Rev. B 102, 094204 (2020) [Editor's suggestion]
- **Decoding the Projective Transverse Field Ising Model**
  Felix Roser, Hans Peter Büchler, and NL
  Phys. Rev. B 107, 214201 (2023)
- **Robust detection of entanglement transitions in the projective transverse-field Ising model**
  Felix Roser, Etienne Springer, Hans Peter Büchler, and NL
  Phys. Rev. Lett. 136, 140403 (2026)
- **Scaling of Entanglement Measures in Loop Models with Extended Criticality**
  Felix Roser, Hans Peter Büchler, and NL
  In preparation (2026)

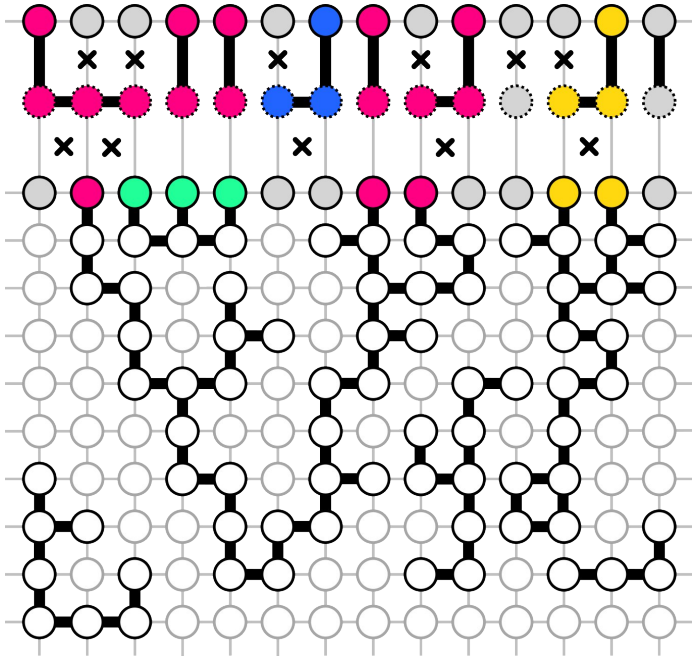
Part II

Engineering quantum matter from scratch

How to construct systems with long-range entangled ground states?



- **Functional completeness of planar Rydberg blockade structures**
  Simon Stastny, Hans Peter Büchler, and NL
  Phys. Rev. B 108, 085138 (2023)
- **Topological order in symmetric blockade structures**
  Tobias F. Maier, Hans Peter Büchler, and NL
  PRX Quantum 6, 030340 (2025) [Featured in Physics]
- **Quantum doubles in symmetric blockade structures**
  Hans Peter Büchler, Tobias F. Maier, Simon Fell, and NL
  Submitted to Phys. Rev. B (2026)
- **Spectral gap of blockade Hamiltonians with Z2 topological order**
  Simon Fell, Tobias F. Maier, Hans Peter Büchler, and NL
  In preparation (2026)



Part I

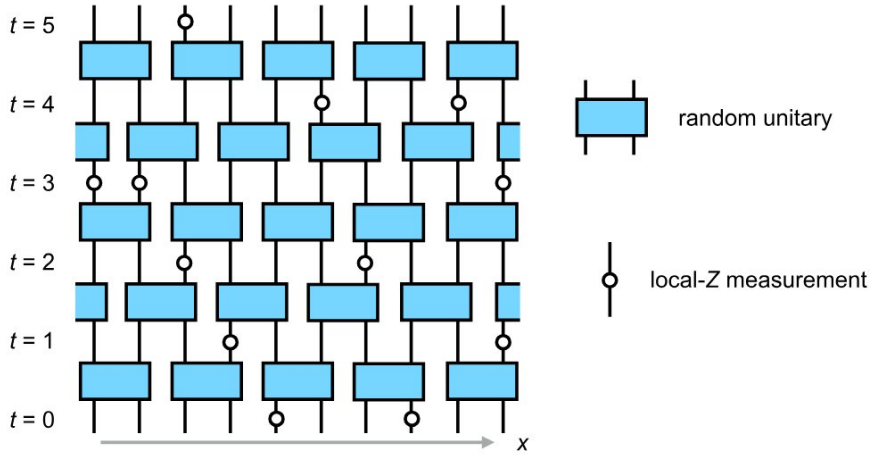
Entanglement Phase Transitions in Monitored Quantum Systems

Can entanglement survive projective measurements?

Motivation: Hybrid Quantum Circuits

Ensembles of quantum circuits with

- **random unitaries**
- **random measurements**

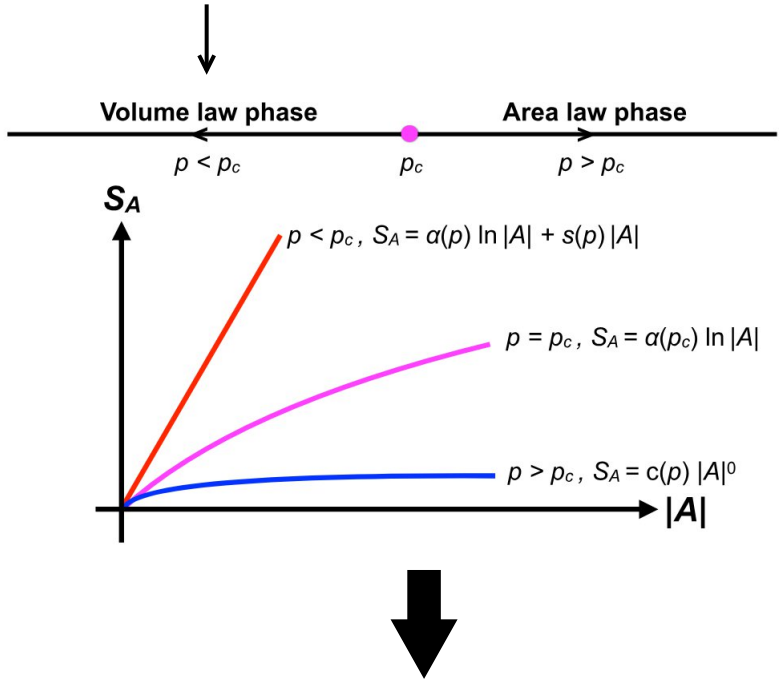


Ensemble average of

Entanglement entropy

$$S_A = -\operatorname{tr}\left[\rho_A \log_2(\rho_A)\right]$$

Entanglement survives projective measurements



Entanglement transition

Measurement-driven entanglement transition in hybrid quantum circuits
Yaodong Li, Xiao Chen, and M. P. A. Fisher, Phys. Rev. B 100, 134306 (2019)

The Projective Transverse-Field Ising Model

Measurement-only
entanglement transition?

Two **non-commuting** measurements:

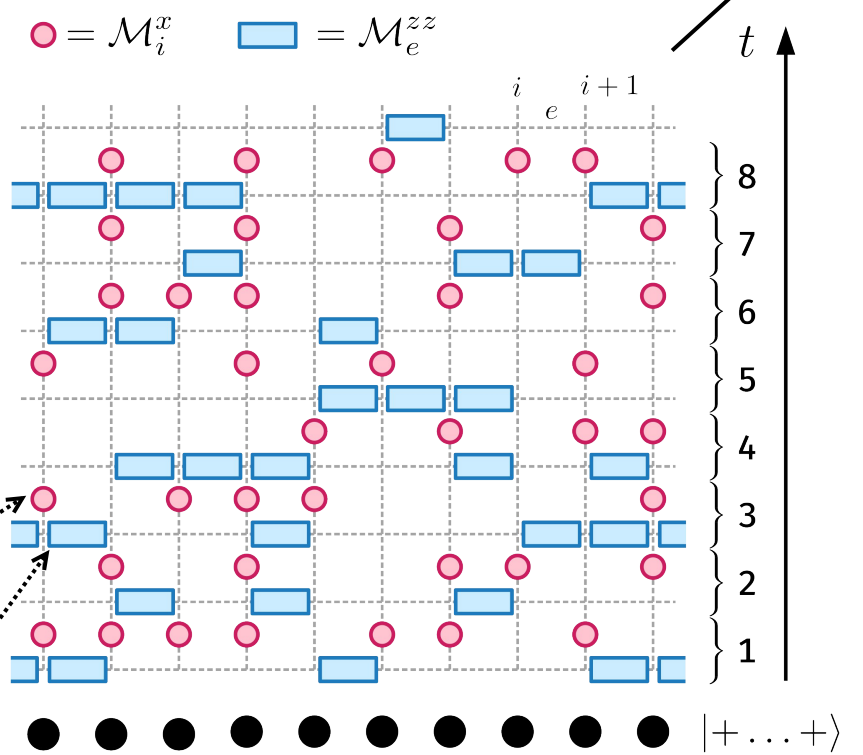
Probability p :

$$\mathcal{M}_i^x = \tfrac{1}{2}(\mathbb{1} \pm X_i)$$

Probability $1-p$:

$$\mathcal{M}_e^{zz} = \tfrac{1}{2}(\mathbb{1} \pm Z_i Z_{i+1})$$

Measurement outcome



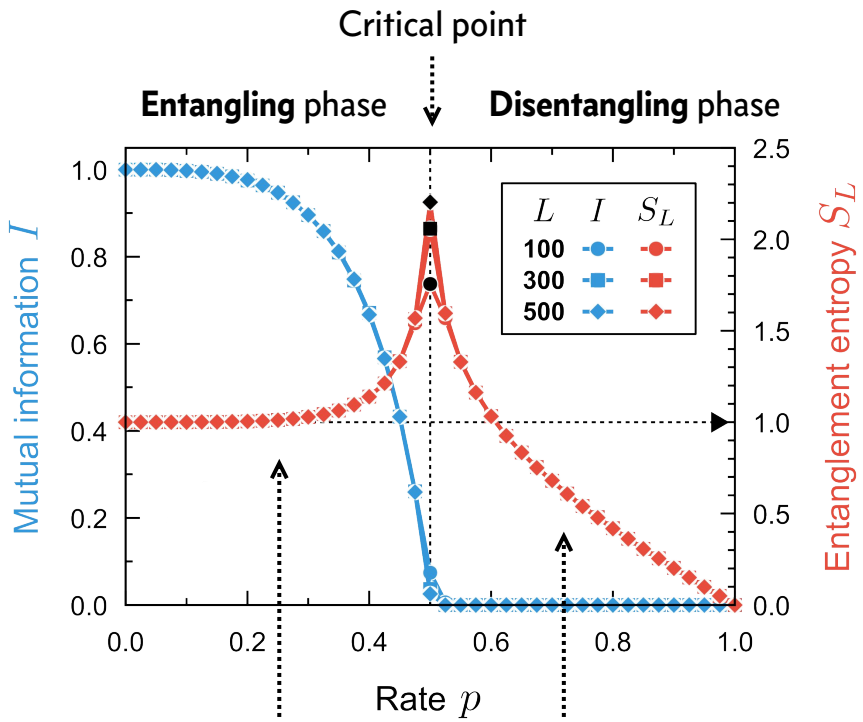
Ensemble of final states (?)

Product initial state

Projective
Transverse-field Ising Model
(PTIM)

Entanglement Transition in the Projective Transverse Field Ising Model
NL and Hans Peter Büchler, Phys. Rev. B 102, 094204 (2020)

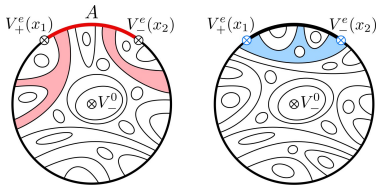
Entanglement Phase Transition



Extended Bell state

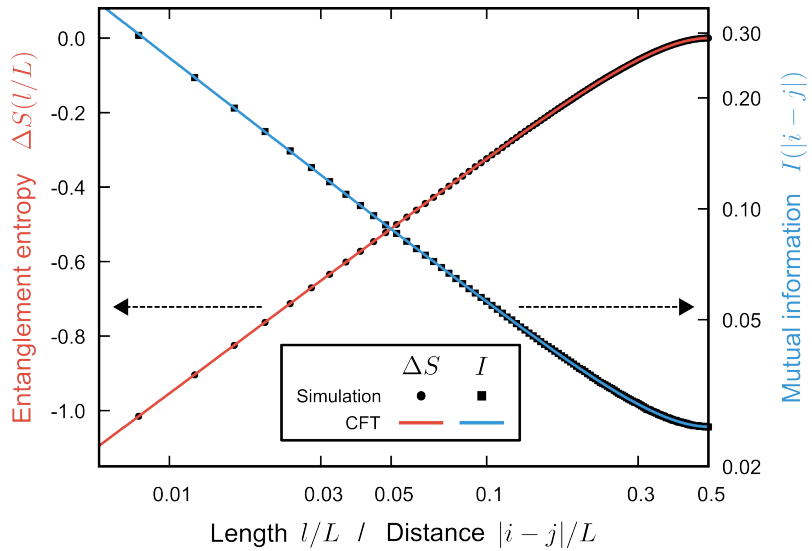
Local Bell states $\frac{1}{\sqrt{2}}(|\mathrm{m}\rangle \pm |\overline{\mathrm{m}}\rangle)$

Entanglement transition at $p_c = 0.5$ between **two area laws**



Describe critical point by **conformal field theory**

(Bond percolation)



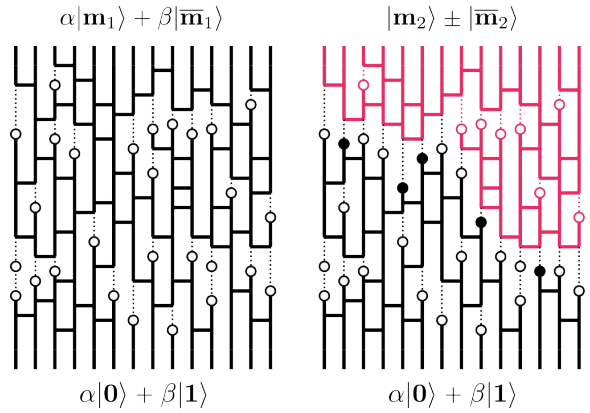
Perfect agreement between **theory** and **numerics**!

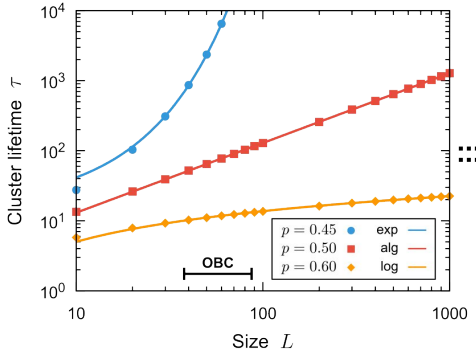
Entanglement Transition in the Projective Transverse Field Ising Model
NL and Hans Peter Büchler, Phys. Rev. B 102, 094204 (2020)

The PTIM and Quantum Error Correction

Retrieve **qubit** from final state →

Store **qubit** in initial state →

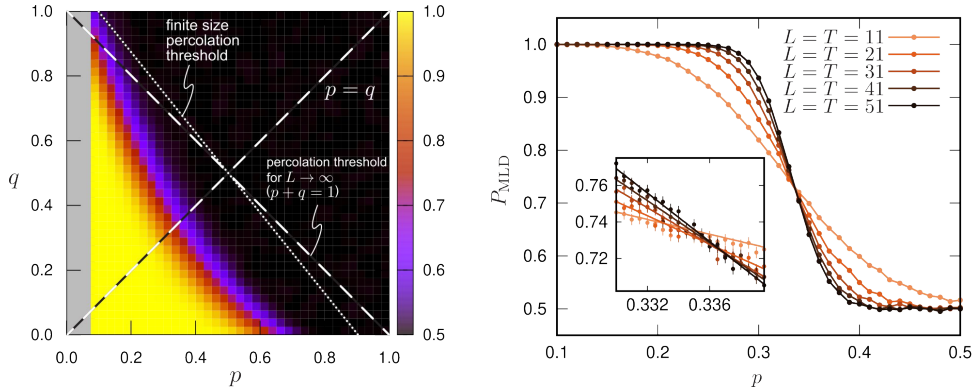




Disentangling phase
→ Qubit **decoheres**

Entangling phase
→ Qubit **survives**

Use
Quantum Error Correction Algorithms
to extract information from final state



Entanglement Transition in the Projective Transverse Field Ising Model
NL and Hans Peter Büchler, Phys. Rev. B 102, 094204 (2020)

Decoding the Projective Transverse Field Ising Model
Felix Roser, Hans Peter Büchler, and NL, Phys. Rev. B 107, 214201 (2023)

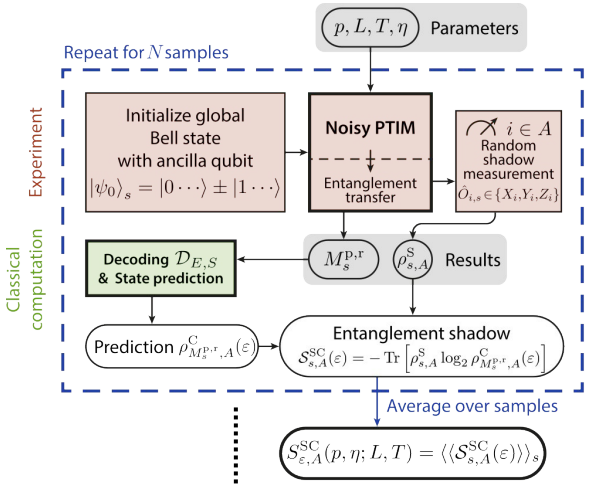
Detecting the PTIM Transition

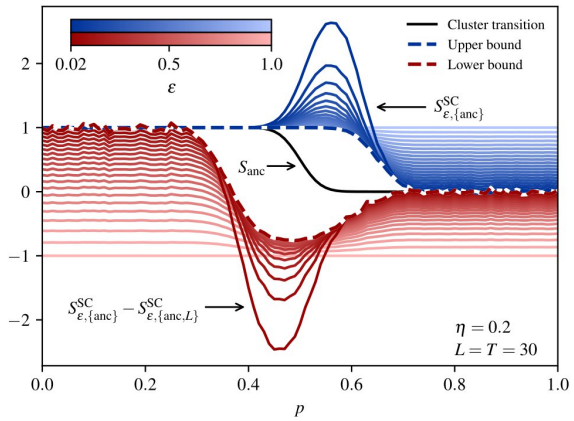
- Randomness of **measurement outcomes**
- **Non-linearity** of entanglement entropy

Postselection problem

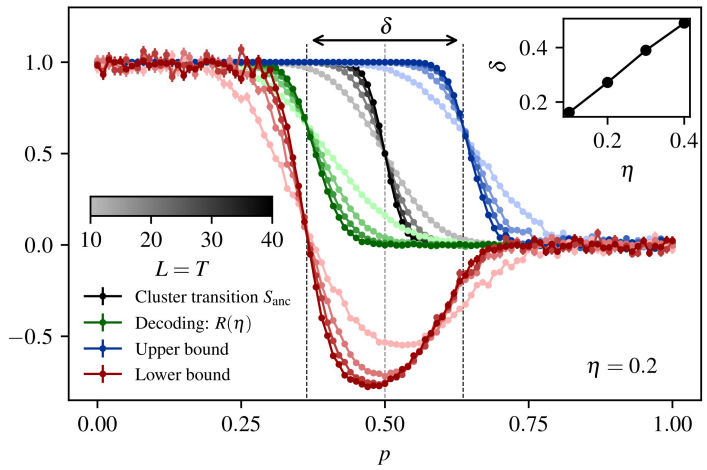
Error rate of measurement record

(Partial) Solution:





Upper and Lower bounds



Measurable & Robust **Upper** and **Lower bounds**

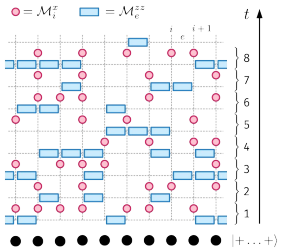
Measurement protocol & Classical post-processing

(Shadow tomography + Klein inequality + Error correction algorithms)

Robust detection of entanglement transitions in the projective transverse-field Ising model
Felix Roser, Etienne Springer, Hans Peter Büchler, and NL, Phys. Rev. Lett. 136, 140403 (2026)

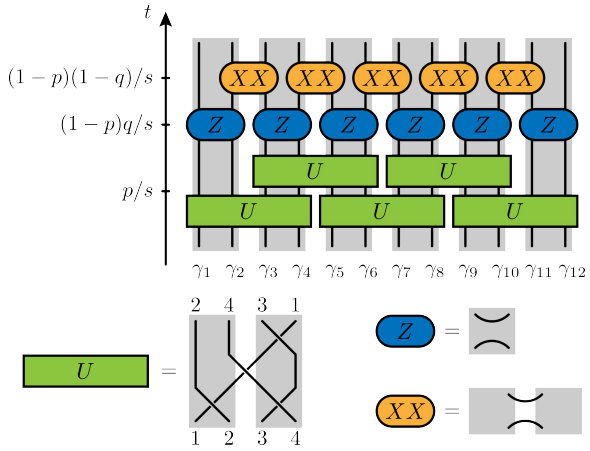
The PTIM and Extended Criticality

Generalize PTIM:



Add **unitaries**

(Cliffords & Matchgates)



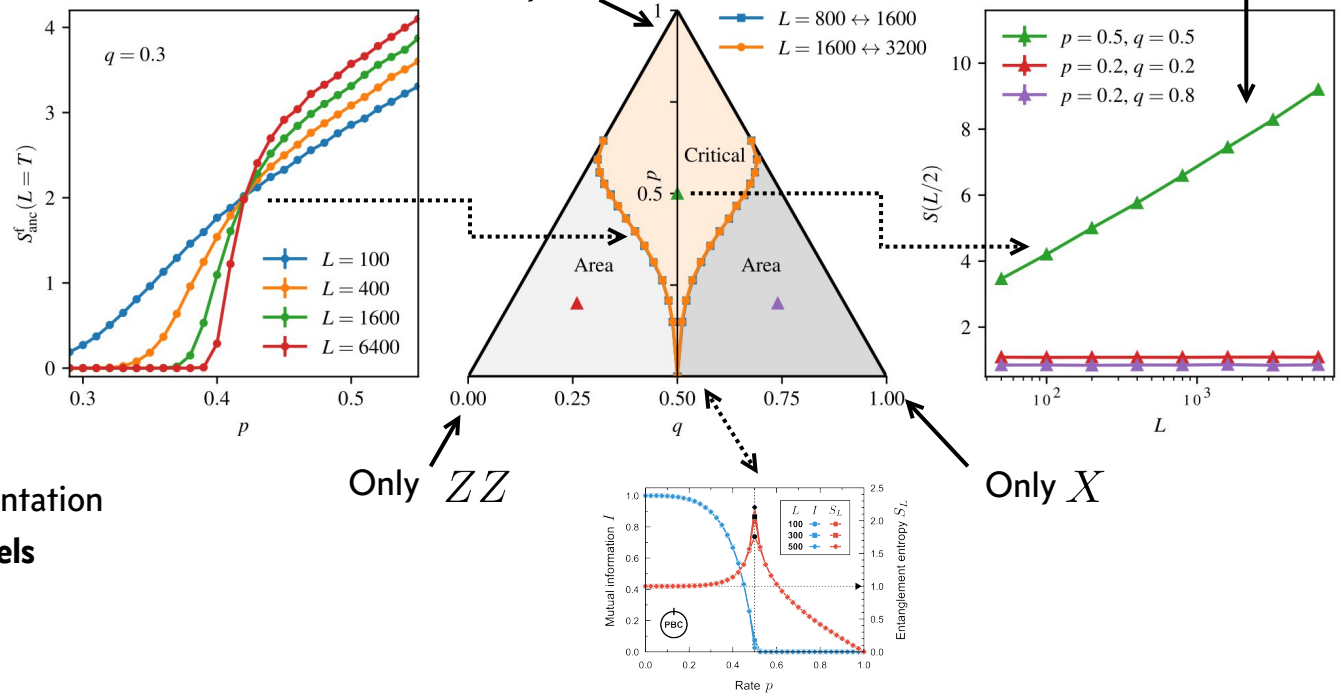
Majorana representation

→ **Loop models**

Extended critical phase

Extensive
(but slowly growing)
entanglement

Only U

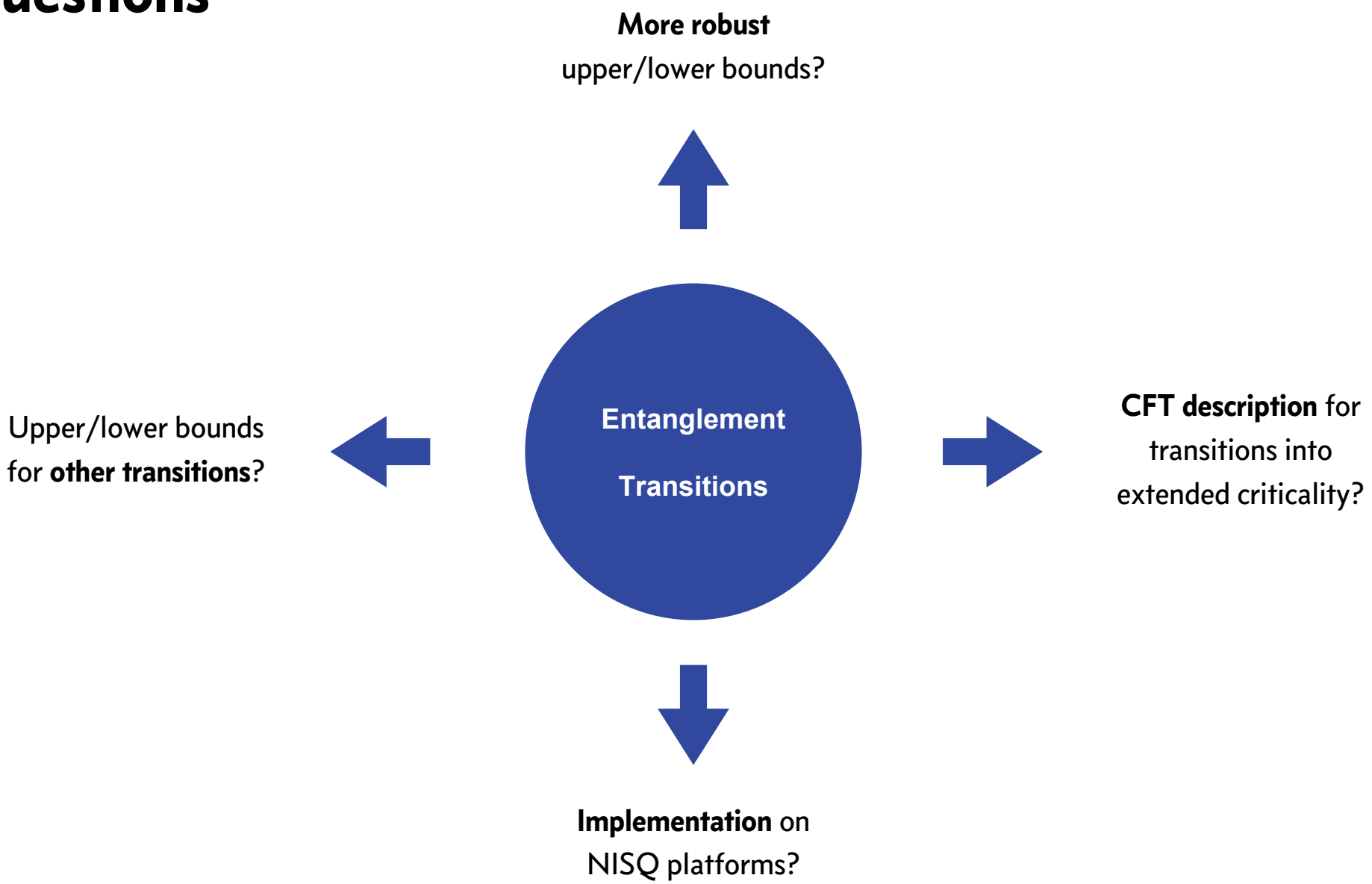


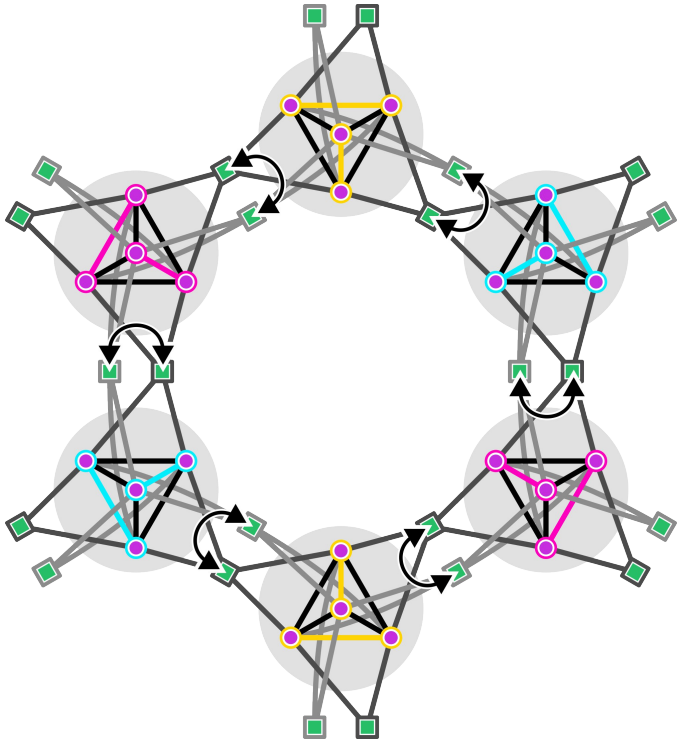
Only ZZ

Only X

Scaling of Entanglement Measures in Loop Models with Extended Criticality
Felix Roser, Hans Peter Büchler, and NL, In preparation (2026)

Open Questions



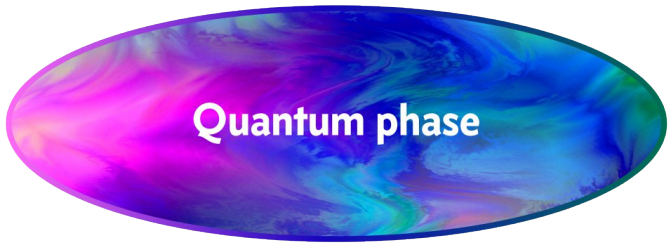


Part II

Engineering Quantum Matter from Scratch

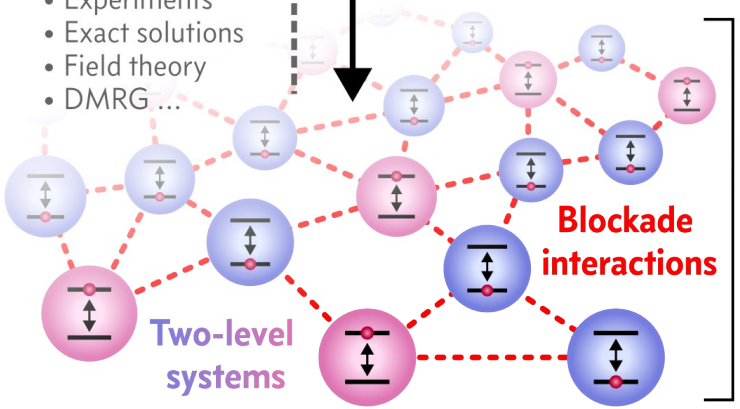
How to construct systems with long-range entangled ground states?

The Inverse Problem of Quantum Many-Body Physics



Emergence
- Experiments
- Exact solutions
- Field theory
- DMRG …

Construction?

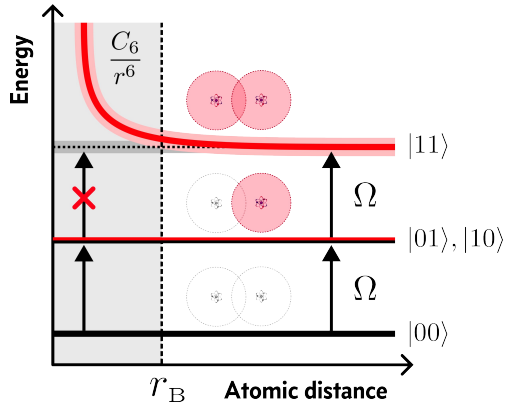


Goal:
→ Complex correlations
→ Long-range entanglement
→ Robust

Toolbox:
→ Two-level systems
→ Blockade interactions
→ Easy to control
→ Potentially noisy

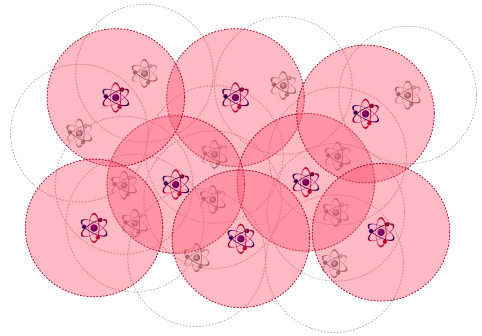
Possible platform:
Rydberg atoms

Strong **van der Waals interaction**



Blockade radius

Rydberg blockade



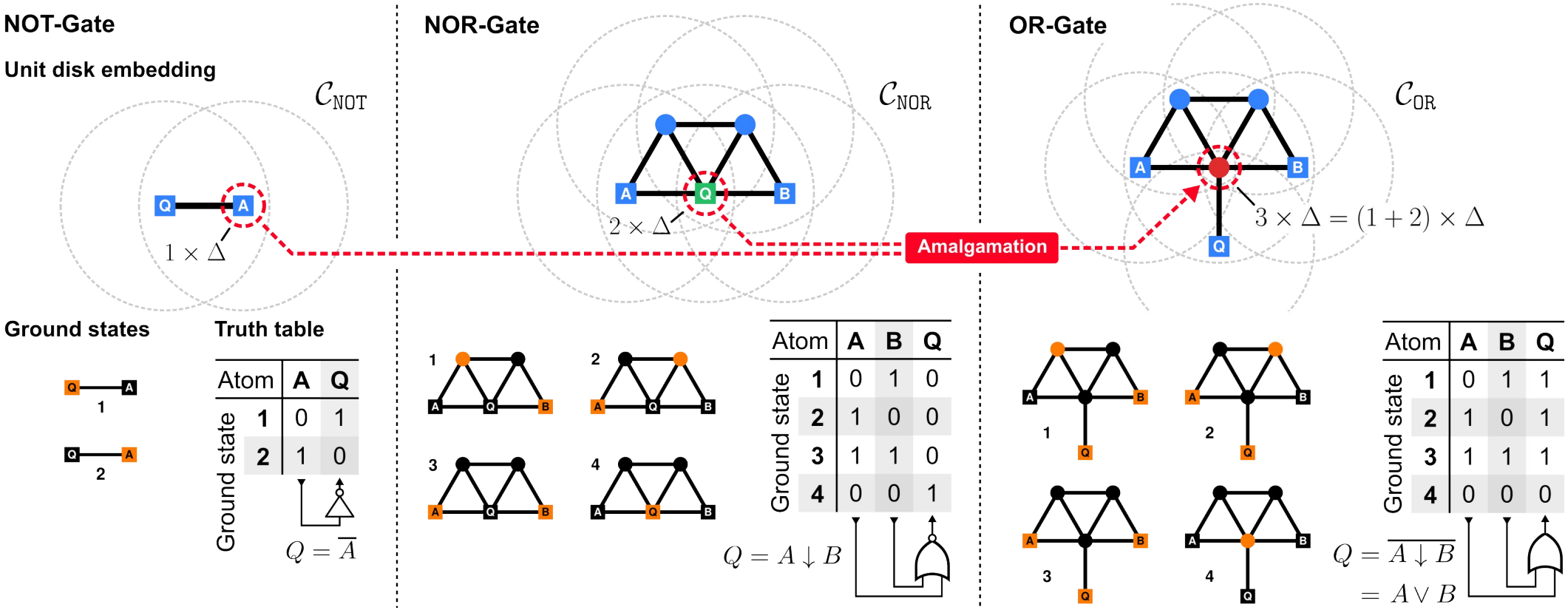
$$H(\Omega) = \sum_{i<j} U(r_{ij})\, n_i n_j + \sum_i (\Omega\, \sigma_i^x - \Delta_i\, n_i)$$

Motivation:
Prediction of Toric Code Topological Order from Rydberg Blockade
Ruben Verresen, M. D. Lukin, and A. Vishwanath, Phys. Rev. X 11, 031005 (2021)

Topological order in symmetric blockade structures
Tobias F. Maier, Hans Peter Büchler, and NL, PRX Quantum 6, 030340 (2025)

Toolbox: Rydberg Blockade Structures

$\Omega = 0$ $\quad \Delta_i = n_i \times \Delta > 0$

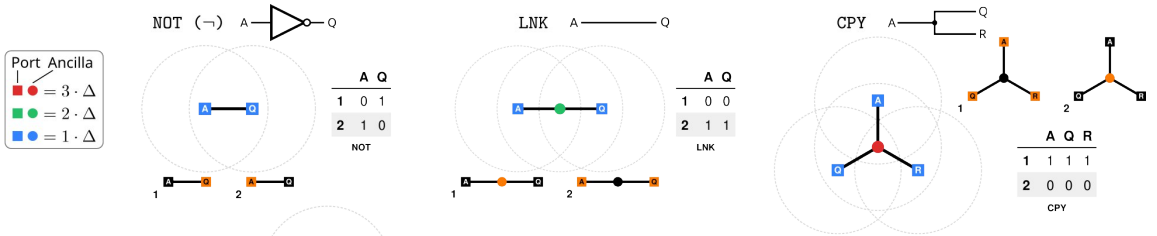


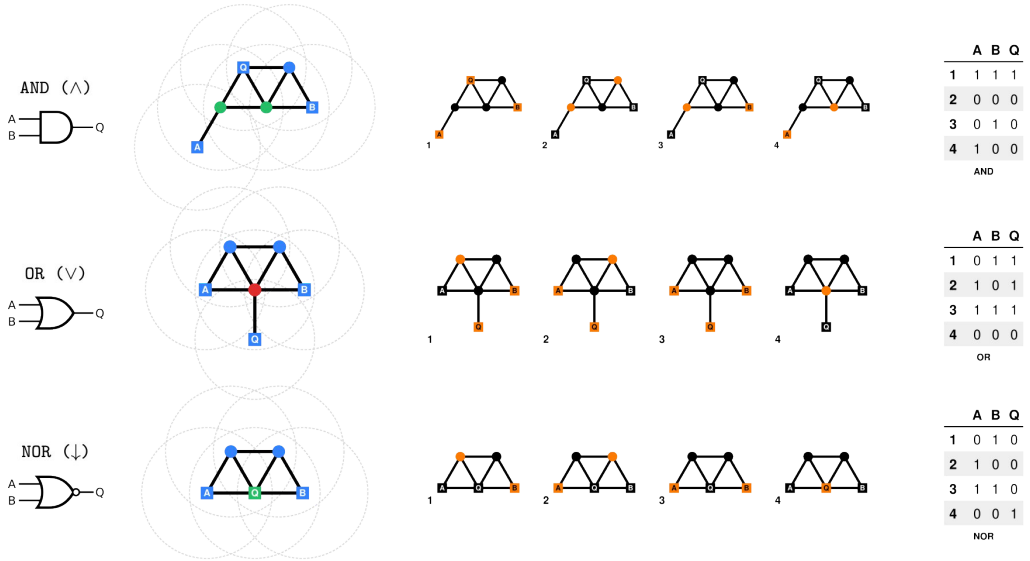
→ Gates can be **combined** into **Boolean circuits**

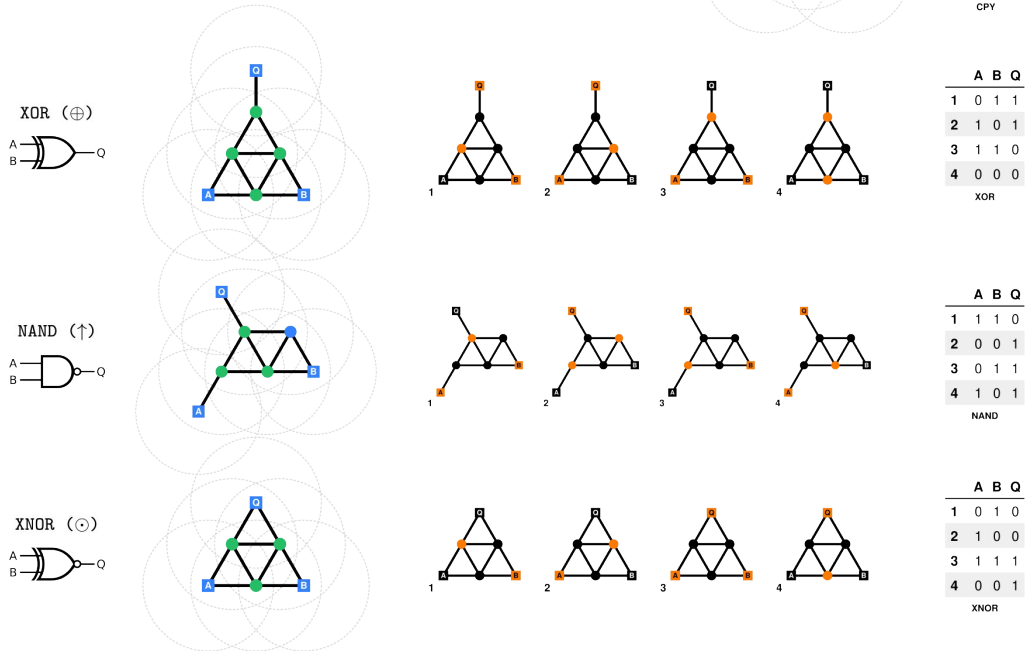
Functional completeness of planar Rydberg blockade structures
Simon Stastny, Hans Peter Büchler, and NL, Phys. Rev. B 108, 085138 (2023)

Functional Completeness

$$\Omega = 0$$







For every **Boolean function** f, there exists a **blockade structure** with **ground state manifold**

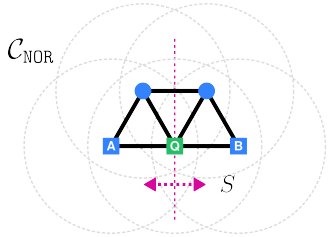
$$\mathcal{H}_f = \mathrm{span}\{\, |\mathbf{n}\rangle \mid f(\mathbf{n}) = 1 \,\}$$

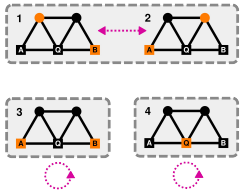
→ Blockade structures are **functionally complete**

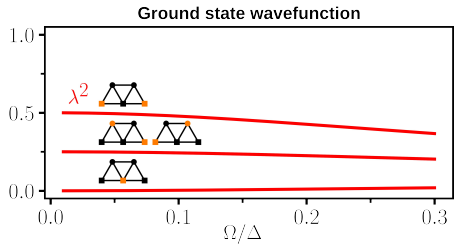
Functional completeness of planar Rydberg blockade structures
Simon Stastny, Hans Peter Büchler, and NL, Phys. Rev. B 108, 085138 (2023)

Quantum Fluctuations and Full Symmetry

Switch on **weak quantum fluctuations:** $0 < \Omega \ll \Delta$

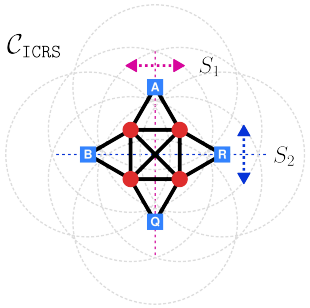


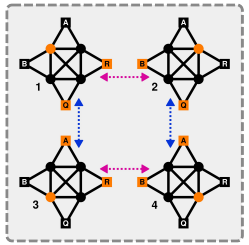


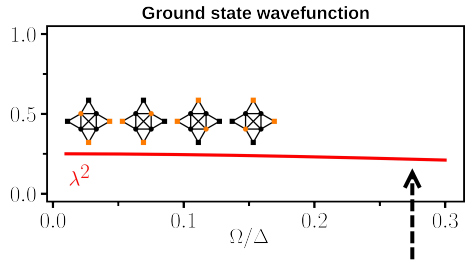


Generic case: **Unique** ground state & **Not** equal-weight

Another example:



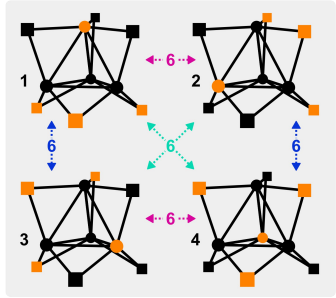




Single orbit ----> **„Fully-symmetric“** ----> **Equal-weight** superposition!

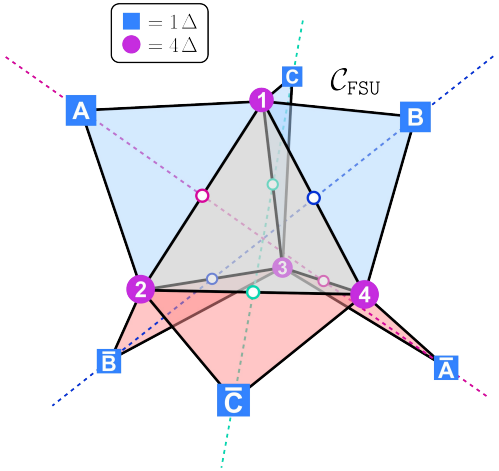
Gate	A (Input)		B (Input)		XOR	XNOR	NOR	$\mathrm{INH}_{\mathrm{AB}}$	AND	$\mathrm{INH}_{\mathrm{BA}}$
Atom	A	$\bar{A}$	B	$\bar{B}$	C	$\bar{C}$	1	2	3	4
Ground state 1	0	1	0	1	0	1	1	0	0	0
Ground state 2	0	1	1	0	1	0	0	1	0	0
Ground state 3	1	0	0	1	1	0	0	0	0	1
Ground state 4	1	0	1	0	0	1	0	0	1	0

Combined **XOR / XNOR** Gates ✓



Fully-symmetric ✓

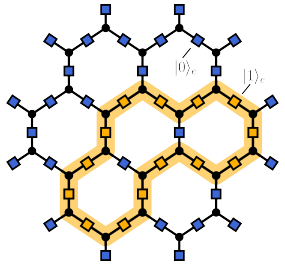
3D Extension



Topological order in symmetric blockade structures
Tobias F. Maier, Hans Peter Büchler, and NL, PRX Quantum 6, 030340 (2025)

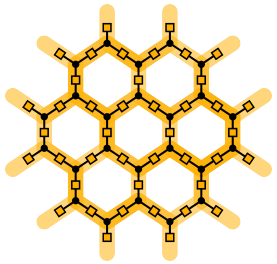
Engineering the Toric Code

Reminder:



Loop states

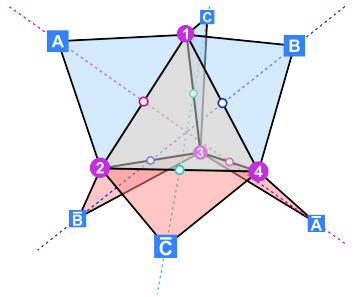
Equal-weight superposition



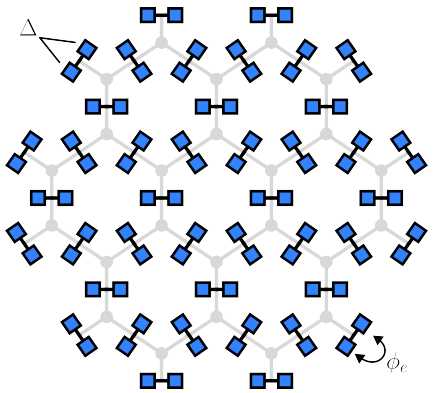
Toric Code ground state

Loop **condensate**

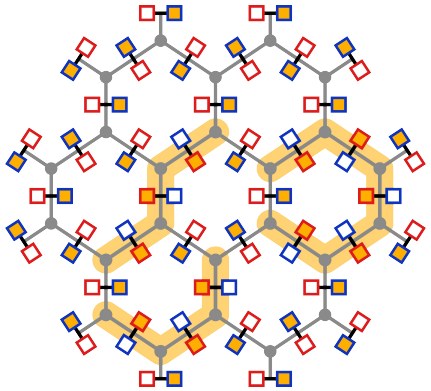
On every **site**:



Toric code as **fully-symmetric blockade structure**?



2 Atoms per edge

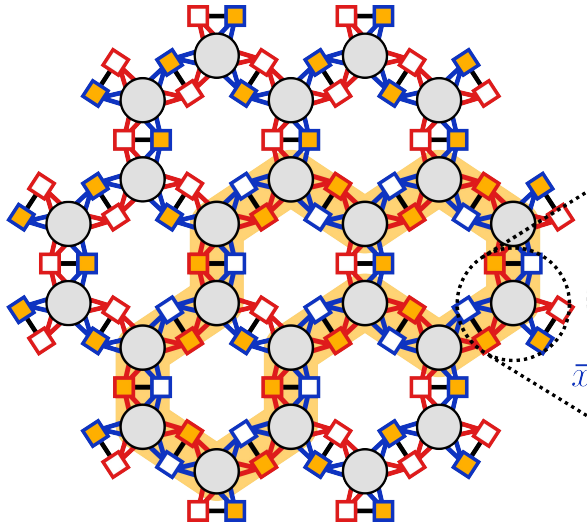


1 Qubit in ground state manifold

(Open strings still allowed!)

„Strings cannot terminate“

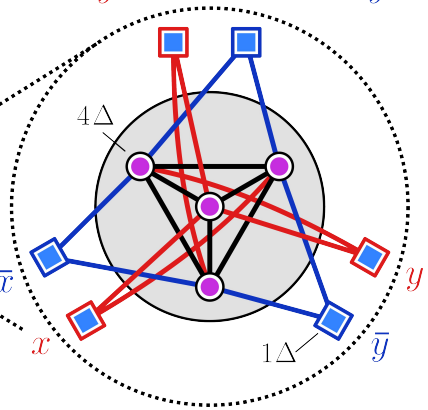
Add **XOR / XNOR** site constraints



$\Omega = 0$

Ground states = **Loop patterns**

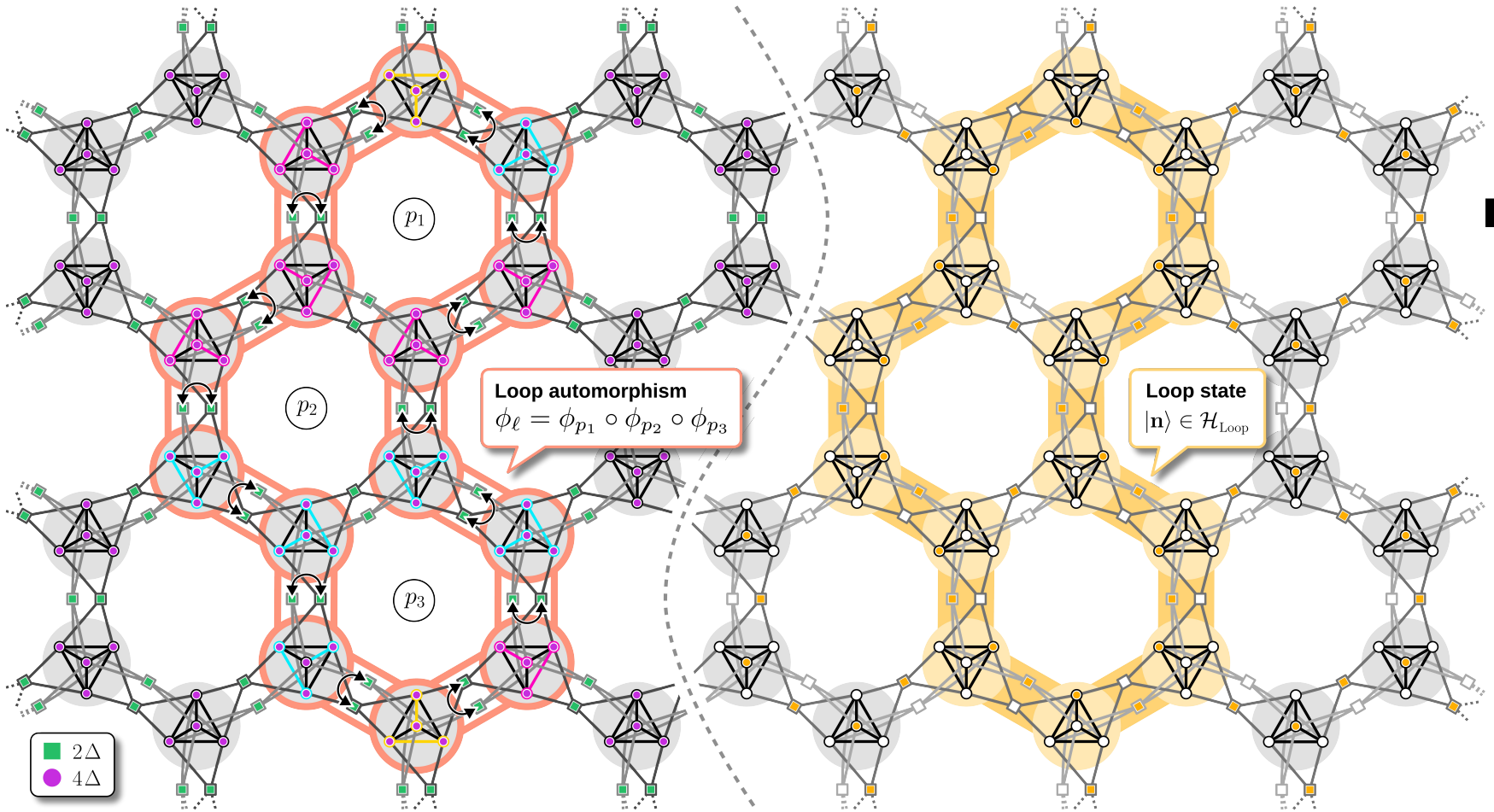
XOR $x \oplus y = z$ XNOR $\bar{z} = \bar{x} \odot \bar{y}$



(Abstract blockades !)

Topological order in symmetric blockade structures
Tobias F. Maier, Hans Peter Büchler, and NL, PRX Quantum 6, 030340 (2025)

Loop Automorphisms and Toric Code Topological Order



By construction:

- Ground states = Loop states ✓
- Fully-symmetric ✓

$0 < \Omega \ll \Delta$

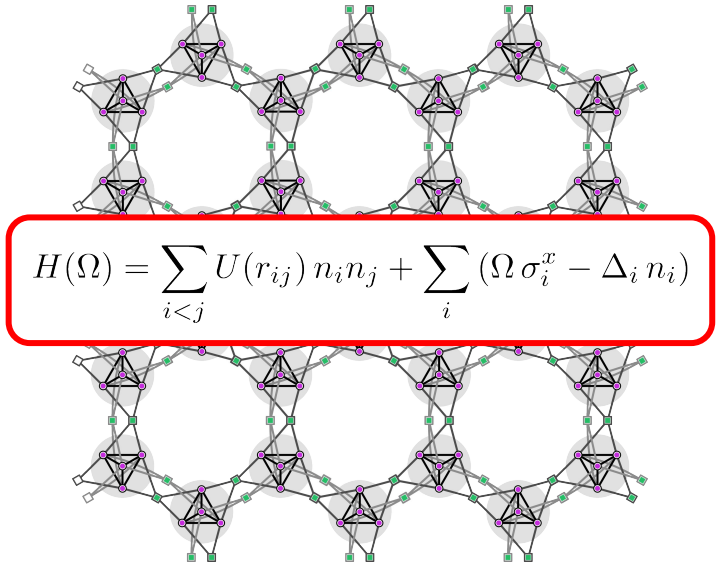
Unique ground state
=
Loop condensate
+ partially depleted states

Gap stability

Ground state topological order

Topological order in symmetric blockade structures
Tobias F. Maier, Hans Peter Büchler, and NL, PRX Quantum 6, 030340 (2025)

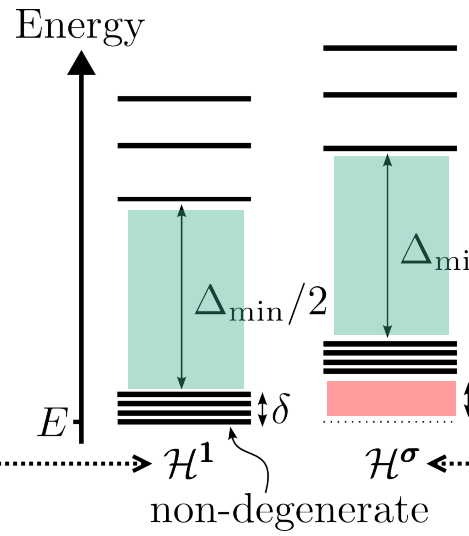
Spectral Gap



$$H(\Omega) = \sum_{i<j} U(r_{ij})\, n_i n_j + \sum_i (\Omega\, \sigma_i^x - \Delta_i\, n_i)$$

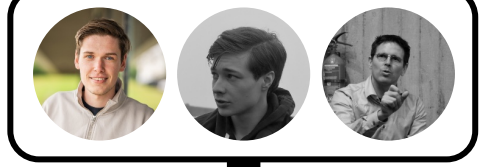
Gapped phase?

Many-body spectrum

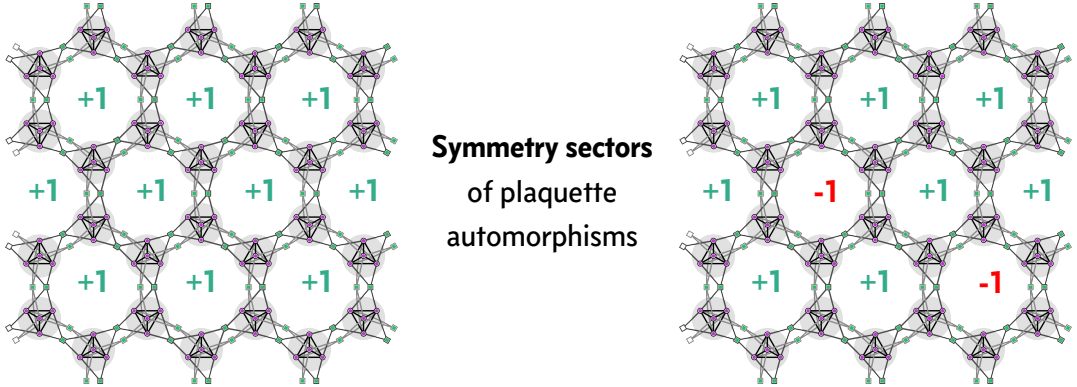


Flux gap → „easy“ to prove ✓

Charge gap → hard to prove …



Gapped phase!



Symmetry sectors of plaquette automorphisms

Ground state in **symmetric sector**

Spectral gap of blockade Hamiltonians with Z2 topological order
Simon Fell, Tobias F. Maier, Hans Peter Büchler, and NL, In preparation (2026)

Constructing Non-Abelian Topological Orders

Toric Code

(topological quantum **memory**)

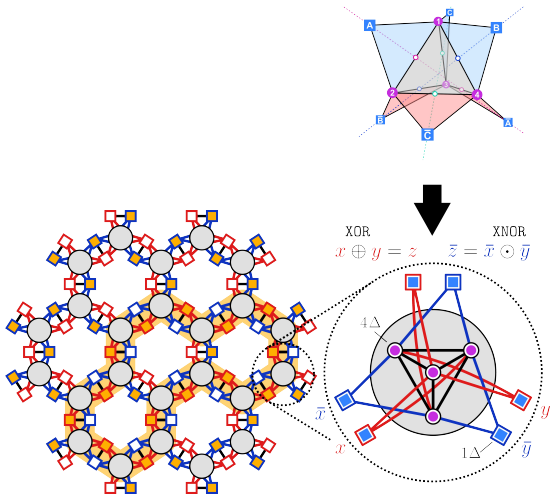
Generalization

Quantum Double for discrete group G

G Abelian → Abelian top. order

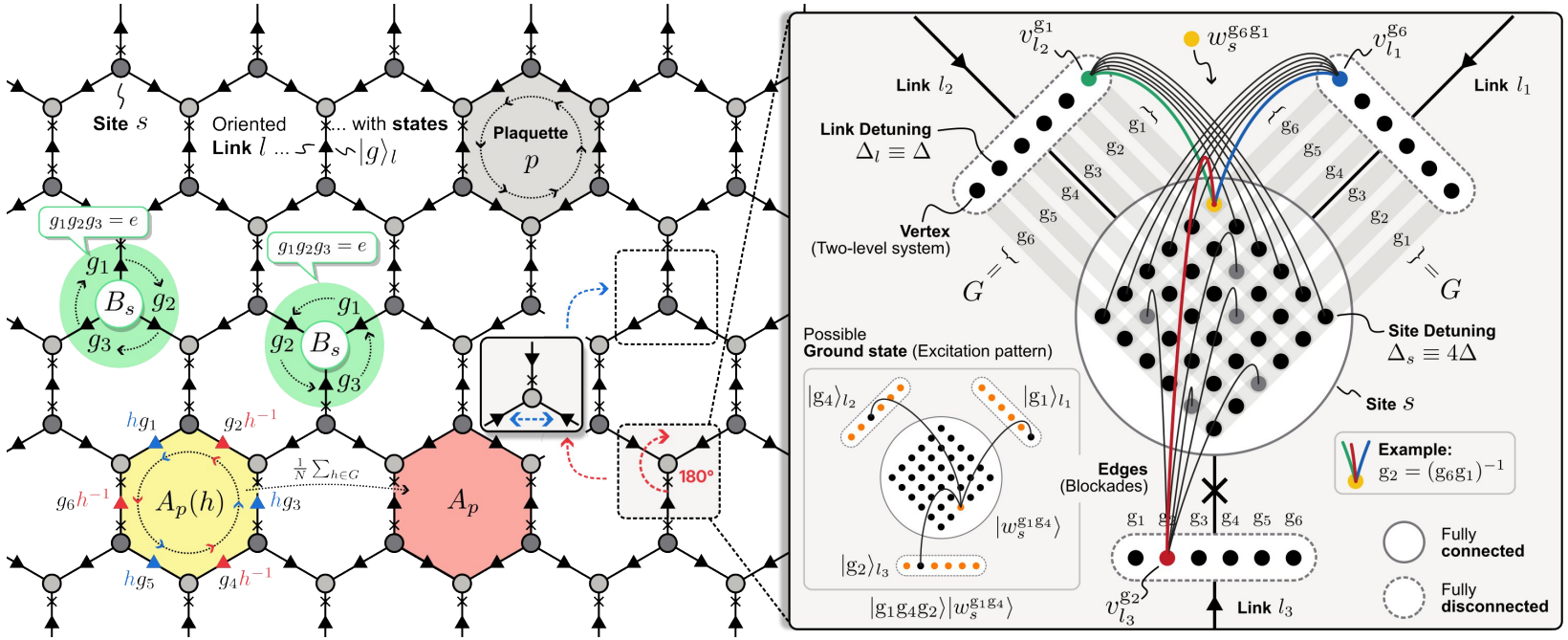
G Non-abelian → **Non-abelian top. order**

(topological quantum **computer**)



Construction for $G = \mathbb{Z}_2$

Generalization



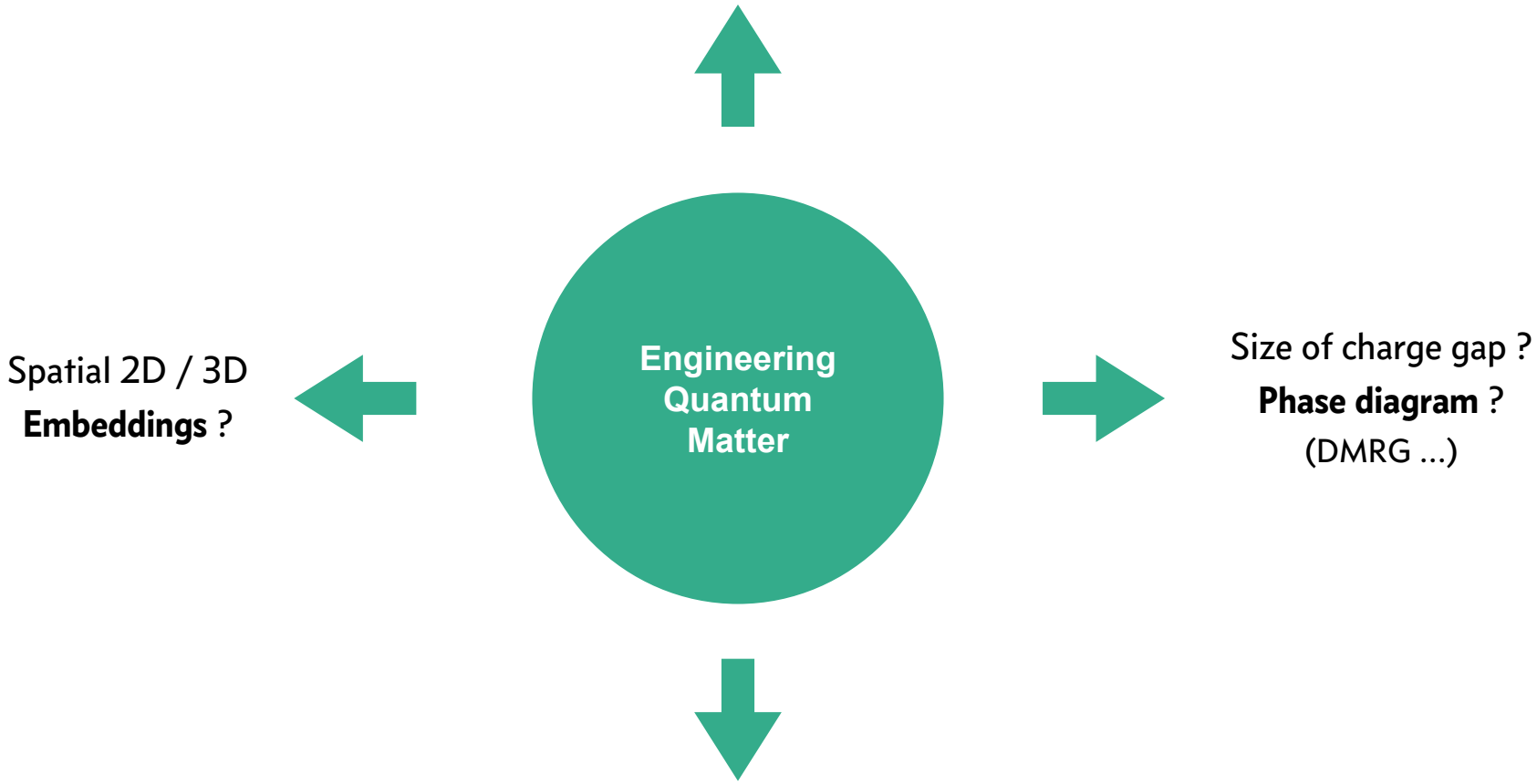
Construction for **arbitrary group** G

Quantum doubles in symmetric blockade structures

Hans Peter Büchler, Tobias F. Maier, Simon Fell, and NL, Submitted to Phys. Rev. B (2026)

Open Questions

Non-equal-weight superpositions ?
(e.g. Fibonacci anyons)



Spatial 2D / 3D
Embeddings ?

Size of charge gap ?
Phase diagram ?
(DMRG ...)

Effects of
van der Waals interactions ?

Summary

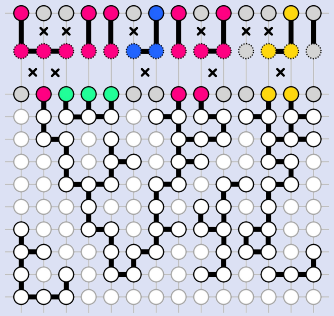
Part I

Entanglement phase transitions in monitored quantum systems

Can entanglement survive projective measurements?

- Quantum circuits with non-commuting projective measurements
- Non-equilibrium phase transition in entanglement structure
- Characterize critical point by conformal field theory
- Explore: Quantum information retrieval, observability, extended criticality ...

- **Entanglement Transition in the Projective Transverse Field Ising Model**
  NL and Hans Peter Büchler
  Phys. Rev. B 102, 094204 [Editor's suggestion] (2020)
- **Decoding the Projective Transverse Field Ising Model**
  Felix Roser, Hans Peter Büchler, and NL
  Phys. Rev. B 107, 214201 (2023)
- **Robust detection of entanglement transitions in the projective transverse-field Ising model**
  Felix Roser, Etienne Springer, Hans Peter Büchler, and NL
  Phys. Rev. Lett. 136, 140403 (2026)
- **Scaling of Entanglement Measures in Loop Models with Extended Criticality**
  Felix Roser, Hans Peter Büchler, and NL
  In preparation (2025)



Part II

Engineering quantum matter from scratch

How to construct systems with long-range entangled ground states?

- Goal: Prepare topological order = Long-range entanglement
- How to realize as many-body ground state with accessible two-body interactions?
- Rydberg blockade structures: functionally complete
- Fully-symmetric blockade structures: Stabilize equal-weight superpositions

- **Functional completeness of planar Rydberg blockade structures**
  Simon Stastny, Hans Peter Büchler, and NL
  Phys. Rev. B 108, 085138 (2023)
- **Topological order in symmetric blockade structures**
  Tobias F. Maier, Hans Peter Büchler, and NL
  PRX Quantum 6, 030340 (2025) [Featured in Physics]
- **Quantum doubles in symmetric blockade structures**
  Hans Peter Büchler, Tobias F. Maier, Simon Fell, and NL
  Submitted to Phys. Rev. B, (2026)
- **Spectral gap of blockade Hamiltonians with Z2 topological order**
  Simon Fell, Tobias F. Maier, Hans Peter Büchler, and NL
  In preparation, (2026)